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Standards Of Fundamental Astronomy

SOFA Tools for Earth Attitude

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Version for Fortran programming language

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1 INTRODUCTION

1.1 The SOFA software

SOFA stands for Standards Of Fundamental Astronomy. The SOFA software is a collection of Fortran 77 and ANSI C subprograms that implement official IAU algorithms for fundamental-astronomy computations. At the present time the SOFA software comprises 192 astronomy routines supported by 55 utility (mainly vector/matrix) routines.

The basic documentation for the SOFA collection is terse, consisting of (i) detailed preamble comments in the individual routines and (ii) classified and alphabetic lists of the routine calls. For an important subset of the SOFA routines, namely those concerned with the Earth’s orientation and rotation, the present document supplements the basic documentation with descriptive material and cookbook examples.

1.2 Quick start

Expert readers may wish to start with Examples, Section 5. Anyone already familiar with the elementary concepts can safely omit the introductory material in Section 2. Those interested in only the latest models can also omit Section 3, which has a large historical component, and start with Current Models, Section 4.

1.3 Abbreviations

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<tr>
<td>BCRS</td>
<td>Barycentric Celestial Reference System</td>
</tr>
<tr>
<td>CIO</td>
<td>Celestial Intermediate Origin</td>
</tr>
<tr>
<td>CIP</td>
<td>Celestial Intermediate Pole</td>
</tr>
<tr>
<td>CIRS</td>
<td>Celestial Intermediate Reference System</td>
</tr>
<tr>
<td>EE</td>
<td>equation of the equinoxes</td>
</tr>
<tr>
<td>EMB</td>
<td>Earth-Moon barycenter</td>
</tr>
<tr>
<td>EO</td>
<td>equation of the origins</td>
</tr>
<tr>
<td>FK4</td>
<td>fourth fundamental catalog</td>
</tr>
<tr>
<td>FK5</td>
<td>fifth fundamental catalog</td>
</tr>
<tr>
<td>GCRS</td>
<td>Geocentric Celestial Reference System</td>
</tr>
<tr>
<td>GMST</td>
<td>Greenwich mean sidereal time</td>
</tr>
<tr>
<td>GST</td>
<td>Greenwich (apparent) sidereal time</td>
</tr>
<tr>
<td>IAU</td>
<td>International Astronomical Union</td>
</tr>
<tr>
<td>ICRS</td>
<td>International Celestial Reference System</td>
</tr>
<tr>
<td>IERS</td>
<td>International Earth rotation and reference systems service</td>
</tr>
<tr>
<td>ITRS</td>
<td>International Terrestrial Reference System</td>
</tr>
<tr>
<td>J2000.0</td>
<td>2000 January 1.5 (in some specified time scale)</td>
</tr>
<tr>
<td>NPB</td>
<td>nutation, precession and frame bias</td>
</tr>
<tr>
<td>SOFA</td>
<td>Standards of Fundamental Astronomy</td>
</tr>
<tr>
<td>ST</td>
<td>sidereal time</td>
</tr>
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<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>TIO</td>
<td>Terrestrial Intermediate Origin</td>
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</tr>
<tr>
<td>TT</td>
<td>Terrestrial Time</td>
</tr>
<tr>
<td>UT</td>
<td>Universal Time UT1</td>
</tr>
<tr>
<td>UTC</td>
<td>Coordinated Universal Time</td>
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<tr>
<td>VLBI</td>
<td>very long baseline interferometry</td>
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2 CELESTIAL COORDINATES

2.1 Stellar directions

Specifying the geometric or apparent direction to a star (or some other body) involves a number of factors: what type of coordinates, the reference triad (the choice of pole and longitude zero), what sort of direction, where the observer is located. Key parts of the SOFA software concentrate on the first and second of these, providing definitive implementations of IAU models for precession-nutation and Earth rotation, supported by tools for manipulating spherical and vector coordinates. SOFA additionally provides a practical level of support for gravitational deflection, aberration, diurnal parallax, refraction and so on – see the SOFA Astrometry Tools cookbook.

SOFA’s transformation capabilities include not only the latest and most accurate methods but also a selection of former standards. Figures 1 and 2 show how some of the systems of celestial coordinates are related, to one another and to the direction in which a celestial source actually appears in the sky. Figure 1 is for the classical system of \textit{mean places}, found in star catalogs and papers prior to the introduction of the International Celestial Reference System (ICRS); Figure 2 begins with ICRS coordinates and represents modern practices. At the bottom of each Figure is the \textit{observed} \([Az, El]\), where a perfect theodolite would be pointed to see the source; and in the bodies of the two diagrams are the intermediate processing steps and coordinate systems.

To help understand these diagrams, and the supporting SOFA routines, we will review the coordinate systems involved, the astronomical phenomena that affect them and the different ways in which celestial coordinates can be managed.

2.2 Precession-nutation

We recall that the familiar \textit{right ascension} and \textit{declination}, \([\alpha, \delta]\), are the names of the longitude and latitude coordinates in a spherical polar system based on the Earth’s axis of rotation. The (classical) zero point of \(\alpha\) is the intersection of the \textit{celestial equator} and the \textit{ecliptic} (the apparent path of the Sun through the year) where the Sun moves into the northern hemisphere. This point is called the \textit{first point of Aries}, or the \textit{equinox}. This simple picture is complicated by the influence on the Earth’s orbit and orientation of the gravitational fields of the solar-system bodies: as a result of these influences neither the equator nor the ecliptic is fixed with respect to the distant background, and consequently a star’s \([\alpha, \delta]\) coordinates are constantly changing.

The slow and large-scale effects are classically referred to as \textit{precession}.

The phenomenon is further divided into \textit{precession of the equator} and \textit{precession of the ecliptic}, corresponding with the classical terms “luni-solar precession” which comes from from the motion of the equator, and “planetary precession” which comes from the motion of the ecliptic.

The precession of the equator is caused by the torque exerted on the distorted and spinning Earth by tidal forces from the Moon (mainly) and Sun, with tiny additional contributions from
the planets. The result is that the Earth’s rotation axis sweeps out a cone centered on the
ecliptic pole, completing one revolution in about 26,000 years.

The precession of the ecliptic is caused by the influence on the Earth’s orbital motion of the
tidal forces from the planets. The effect is a slow (0\′′.5 per year) secular rotation of the ecliptic
about a slowly-moving diameter.

The ever-changing gravity gradient in which the Earth is immersed leads to a precessional motion
that is itself constantly varying. However, the main 26,000-year component is on such a grand
scale that it dwarfs the remaining terms, the biggest of which has an amplitude of only 11\′′ and
a period of about 18.6 years. This difference of scale makes it convenient to treat these two
components of the motion separately, the smaller part being called nutation. Thus precession
and nutation are simply different frequency components of a single physical phenomenon.

The models for precession-nutation come from a combination of observation and theory, and
are subject to continuous refinement. In recent decades, nutation models in particular have
become quite elaborate; the IAU 1980 nutation model provided by the SOFA routine iau_NUT80
comprises only 106 terms in each of $\psi$ (longitude) and $\epsilon$ (obliquity), from 100\′′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\′\ primed

2.3 Evolution of celestial reference systems

Classical text-books use reference systems that are based on the celestial equator and the ecliptic,
with the equinox as the zero point for right ascension. This simple geometrical picture is
familiar, comfortable and works well for ordinary low-precision applications such as describing
astronomical phenomena. However, when applied to more precise problems, complications arise.
The Earth’s pole (and hence equator) is in constant motion because of precession, nutation and
polar motion; the ecliptic is a somewhat ill-defined concept and is itself constantly changing
because of planetary perturbations. Using these moving planes in high-precision applications
such as VLBI or pulsar timing became extremely complicated, and by the end of the 20th
century the recommended procedures had changed:

1. The International Celestial Reference System was introduced, fixing the coordinate triad
with reference to very distant objects. Although ICRS coordinates are close to FK5 J2000.0
mean places, this alignment was made merely for convenience, providing a degree of con-

continuity. The ICRS is completely independent of the rotation and motion of the Earth: a
given star has ICRS coordinates that are essentially fixed, changing only as a consequence
of the star’s own motion through space. In addition to the ICRS itself, new barycentric
(BCRS) and geocentric (GCRS) celestial reference systems were introduced, to describe
the relationship between different choices of space-time coordinates. These different sys-
tems are intimately related to the changing apparent direction of the light ray from the star
as seen by different observers. As can be seen from Figure 2, BCRS coordinates apply to
the star position once space motion has been taken into account; the apparent direction of
the star as seen by a terrestrial observer is affected by parallax, light deflection by the Sun
(and planets) and aberration, the result being expressed in GCRS coordinates. In effect,
ICRS, BCRS and GCRS share the same orientation, and it is only once precession-nutation
is considered that axes with a different orientation arise.
Prior to the introduction of the International Celestial Reference System, star positions were published or cataloged using one of the mean \([\alpha, \delta]\) systems shown at the top, each with its own specific precession and aberration models. The “FK4” systems, not at present supported by SOFA, were used before 1984 and were usually equinox B1950.0. The “FK5” system, equinox J2000.0 (to which the ICRS is nominally aligned), then took over. The figure relates a star’s mean \([\alpha, \delta]\) to the actual line-of-sight to the star.

Figure 1: Relationship between celestial coordinates, pre-ICRS
Figure 2: Relationship between celestial coordinates, post-IAU-2000
Since the introduction of the ICRS, the moving equator and ecliptic no longer provide the basis of high-precision astrometry, and mean places have fallen out of use. ICRS was nominally aligned to J2000.0 mean, with a residual frame bias of about 23 mas that can be ignored for many applications. In addition, a new zero-point for right ascension of date, the CIO, has replaced the equinox, breaking the final link with the ecliptic as far as the user is concerned. The figure relates a star’s ICRS $[\alpha, \delta]$ to the actual line-of-sight to the star. The right-hand branch shows the modern CIO/ERA method of reckoning the Earth rotation; the classical equinox/GST method, shown in the left-hand branch, remains an option for supporting older applications.
2.4 The IAU 2000 changes

2. The ecliptic was retired as a reference plane for high-precision work. One difficulty had been a certain degree of confusion about exactly what “ecliptic” meant. Classically, the basis was the geometric path of the Sun as seen from the Earth-Moon barycenter, but it had become more usual instead to start from the EMB orbital angular momentum vector. It turns out these give the same result only if the orbital angular momentum vector is referred to the moving ecliptic itself; the angular momentum vector in inertial space leads to a slightly different plane, moving the equinox by about 0′′.1. Other difficulties with the ecliptic included exactly how to average out the orbital perturbations, uncertainty about whether the plane should go through the Sun or the solar-system barycenter, and so on.

3. Greenwich sidereal time (GST), which is a complicated function of both Earth rotation (UT1) and time (not quite the same thing), was replaced by Earth rotation angle (ERA), which is a straightforward linear transformation of UT1 alone and which represents the Earth’s rotation in inertial space.

4. The corollary of the change from GST to ERA is that the zero point of apparent right ascension has changed. The new α zero-point is called the celestial intermediate origin (CIO). The CIO, a non-rotating origin, is kinematically, not geometrically, defined, with the result that it is impossible to draw a diagram that explains where the CIO is on a given date. However, the concept is easy to grasp: the CIO is simply a point on the equator that moves as little as possible. It necessarily bobs north and south as the equator of date moves, but does not move along the equator; from moment to moment its motion is always at right-angles to the equator. For epoch 2000.0 the CIO was placed close to GCRS right ascension zero, and throughout the 20th and 21st centuries it remains within 0′′.1 of that alignment.

2.4 The IAU 2000 changes

The switch from equinox/GST to CIO/ERA, introduced by the IAU in 2000 in order to bring about a clean separation between Earth rotation and precession-nutation, will take some years to come into wide use. Although recent textbooks have embraced the ICRS, all but a handful pre-date the introduction of the CIO, making it difficult to teach the new methods – even though they are in many ways simpler, with less to go wrong. There is in any case little incentive now that astronomers no longer need to prepare apparent places for their observing runs, a chore left to the telescope control computer. Furthermore, the latest and most precise precession-nutation and Earth rotation theories can be presented using either framework, with equal accuracy and with hardly any difference in the total amount of computation required. Given these factors, and taking into account the large number of equinox/GST based applications that exist, the SOFA policy at present is to support both methods – with the strong recommendation that for all new applications the CIO based methods should be adopted.

2.5 Frame bias

The International Celestial Reference System was nominally oriented to match the J2000.0 dynamical frame. Subsequent observations located the J2000.0 mean pole at ICRS \([X, Y]\)
coordinates \([-0.016617, -0.0068192]\)\(^1\), with the J2000.0 mean equinox lying \(0.0146\) west of the ICRS meridian.

### 2.6 CIO and TIO

The celestial intermediate origin, CIO, is located using a quantity called \(s(t)\), which is the difference between two right ascensions of the same point. The point is the ascending node of the (moving) CIP equator on the (fixed) GCRS equator; the two right ascensions are with respect to the GCRS and the CIP/CIO respectively. The quantity \(s\) is defined by an integral that depends on the path of the precessing-nutating CIP, but is normally estimated using a series. The series used in the SOFA routines \texttt{iau\_S00} and \texttt{iau\_S06} is in fact for \(s + XY/2\), where \(X\) and \(Y\) are the CIP coordinates of date.

There is an analogous quantity \(s'\) that applies to terrestrial coordinates, depends on the history of polar motion, and locates the terrestrial intermediate origin, TIO. It is extremely small, under 0.1 mas even after two centuries. The SOFA routine \texttt{iau\_SP00} implements a standard model.

### 2.7 Equation of the origins

The equation of the origins (EO) is a useful quantity that captures the essential difference between the old and the new systems. In applications that need to support both the old, equinox based, and new, CIO based, systems, it will usually be found best to base the application on CIO/ERA and to use the EO when dealing with sidereal time and equinox based right ascensions.

The expression for calculating hour angle is the same in either the old or new system:

\[
h = \theta - \alpha,
\]

where \(\theta\) is now Earth rotation angle but was formerly Greenwich apparent sidereal time, and \(\alpha\) is referred to the celestial intermediate origin but was formerly referred to the true equinox of date. The difference ERA–GST, or equivalently the distance from the equinox to the CIO, is called the “equation of the origins” (EO). The EO is dominated by the precession in right ascension, causing it to change by about \(46''\) per year, but includes also the nutation in right ascension (otherwise known as the equation of the equinoxes) and a number of small correction terms.

### 2.8 Equinox versus CIO

Although in the classical picture precession-nutation is thought of as affecting both the equator and the equinox, the latter aspect is nothing more than a geometrical consequence of the chosen coordinate system. The essence of precession-nutation is that it is all about the motion of the Earth’s pole (and hence the equator), and in the post-IAU-2000 CIO based approach it is both

\(^1\)When components of a unit vector are quoted in angular measure this should be taken to mean the radian equivalent.
normal and sufficient to specify the instantaneous pole, which is called the \textit{celestial intermediate pole} (CIP), simply as \( [X,Y] \), the first two components of the CIP’s GCRS unit vector. Indeed, one attraction of the new method is that series for \( X(t) \) and \( Y(t) \) exist (see the SOFA routine \texttt{iau_XY06} ) that deliver the CIP as a function of time in a particularly straightforward way.

Even so, the ecliptic is never far away. The tidal forces that cause precession-nutation are from bodies that remain near the ecliptic plane, and descriptions of these effects are at their most concise when expressed in ecliptic based coordinates. Hence even in the latest precession theories there is still an ecliptic, and the existing nutation series are with respect to the mean equator and ecliptic of date.

Moreover, the classical precession angles, when used to generate the \( 3 \times 3 \) precession matrix, automatically deliver the equinox, whether it is wanted or not. It appears as the top row of the epoch-to-date precession-nutation matrix, which is the unit vector towards the equinox (just as the bottom row is the unit vector towards the pole; the middle row is simply the vector product pole \( \times \) equinox).

On the face of it, this makes direct use of the classical matrix attractive. Starting with existing precession and nutation models, it delivers traditional equinox based right ascensions without further ado, ready to be used in conjunction with sidereal time. However, there is a hidden danger. Any change in the precession or nutation models requires a corresponding change to the formulation of sidereal time, and it is the SOFA user’s responsibility to employ a matching set of precession, nutation and sidereal time models if hour angles are to be calculated reliably. For example, mixing old and new precession-nutation and sidereal time can introduce errors in \( h \) at the 0’05 level.

In the newer SOFA routines, multiple sidereal time formulations are avoided by working internally with respect to the CIO, which is nearly independent of the adopted precession-nutation model. The distance between the CIO and the chosen equinox is the \textit{equation of the origins} (see Section 2.7). The EO can be calculated from any supplied precession-nutation matrix, and if subtracted from the Earth rotation angle automatically yields a Greenwich apparent sidereal time appropriate for use with that particular matrix.

\section*{2.9 The celestial to terrestrial transformation}

Figure 2 shows how frame bias, precession-nutation, Earth rotation and polar motion form a chain of transformations connecting celestial coordinates (GCRS) to terrestrial coordinates (ITRS). Written out in matrix form, the chain is:

\begin{equation}
\mathbf{v}_{\text{ITRS}} = \mathbf{R}_{\text{PM}} \times \mathbf{R}_3(\theta) \times \mathbf{R}_{\text{NPB}} \times \mathbf{v}_{\text{GCRS}}
\end{equation}

where:

\begin{itemize}
  \item \( \mathbf{v}_{\text{GCRS}} \) and \( \mathbf{v}_{\text{ITRS}} \) are the same direction with respect to the two reference systems;
  \item the matrix \( \mathbf{R}_{\text{NPB}} \) represents the combined effects of frame bias and precession-nutation and defines the orientation of the celestial intermediate pole and a longitude origin;
\end{itemize}
\[ \mathbf{R}_3(\theta) \] is Earth rotation, with \( \theta \) either ERA or GST depending on whether \( \mathbf{R}_{\text{NPB}} \) is CIO based or equinox based; and

- the matrix \( \mathbf{R}_{\text{PM}} \) takes account of polar motion.

The sequence of coordinate systems in Eq. (1) is:

1. The Geocentric Celestial Reference System, which is effectively the same orientation as ICRS. The bias+precession+nutation transformation then produces...

2. ...either Celestial Intermediate Reference System coordinates, if the matrix \( \mathbf{R}_{\text{NPB}} \) was CIO based, or classical apparent place, if the matrix \( \mathbf{R}_{\text{NPB}} \) was equinox based. The rotation through either ERA or GST, as appropriate, then gives...

3. ...Terrestrial Intermediate Reference System coordinates, which after polar motion has been applied and the small adjustment onto the terrestrial intermediate origin (TIO) has been made gives...

4. ...International Terrestrial Reference System coordinates: where on the Earth the star appears to be in the (geographical) zenith, and easily transformed into \([h, \delta]\) for other locations.

### 3 SOFA EARTH ATTITUDE MODELS

#### 3.1 Classical precession

For equinox based use, SOFA offers a choice of three models for the classical precession:

- The Fricke-Lieske, IAU 1976, “FK5” model. The conventional \( \zeta, z, \) and \( \theta \) Euler angles are generated by the SOFA routine \texttt{iau\_PREC76} and the precession matrix by \texttt{iau\_PMAT76}.

- The IAU 2000 model. This is in the form of corrections, due to Mathews, Herring & Buffet, to the IAU 1976 model. The corrections can be obtained by calling \texttt{iau\_PR00} (rates) and \texttt{iau\_BI00} (position, i.e. frame bias). The complete precession matrix, including frame bias, is available from \texttt{iau\_PMAT00}, while individual frame bias and precession matrices can be obtained with \texttt{iau\_BP00}.

- The Capitaine et al. IAU 2006 model. The SOFA routine \texttt{iau\_PFW06} produces the four Fukushima-Williams angles, \( \bar{\gamma}, \bar{\phi}, \bar{\psi} \) and \( \epsilon_A \), that automatically include the frame bias. The precession+bias matrix is available from \texttt{iau\_PMAT06}. Another advantage of the Fukushima-Williams parameterization is that the classical nutation angles \( \Delta \psi \) and \( \Delta \epsilon \) can be added directly to the final two angles, \( \bar{\psi} \) and \( \epsilon_A \), if the desired end product is the complete nutation+precession+bias matrix.

Another aspect of precession is the obliquity of the ecliptic. There are two supported models:

- The IAU 1976 mean obliquity, which can be computed by calling \texttt{iau\_OBL80}. This obliquity also forms the foundation for the IAU 2000A/B precession-nutation.

- The IAU 2006 mean obliquity, \texttt{iau\_OBL06}.


3.2 Classical nutation

SOFA provides three nutation models:

- The IAU 2000A model: `iau_NUT00A`.
- The IAU 2000B model: `iau_NUT00B`.

All three return the classical components of nutation in longitude and obliquity, $\Delta \psi, \Delta \epsilon$, from which a nutation matrix can (if required) be generated by calling `iau_NUMAT`, and from which the equation of the equinoxes (EE), described later, can be found.

The IAU 2000B model is an abridged version of the full 1365-frequency IAU 2000A model. It is a fraction of the size (smaller even than the IAU 1980 model) and achieves 1 mas accuracy throughout 1995-2050.

The best nutation model that is currently available is IAU 2000A. When using this in conjunction with the latest (IAU 2006) precession model it is formally necessary to apply certain small adjustments to the nutation. This IAU-2006-compatible form of the IAU 2000A nutation is available from the routine `iau_NUT06A`. When using the IAU 2000B nutation these adjustments can be neglected.

If the application requires the nutation in the classical matrix form, it can be generated in a single call by using `iau_NUTM80`, `iau_NUM00A`, `iau_NUM00B` or `iau_NUM06A` for the IAU 1980, IAU 2000A, IAU 2000B or IAU 2006/2000A variants respectively.

3.3 The CIP $X,Y$

SOFA offers six routines for obtaining $X,Y$, the first two coordinates of the CIP unit vector:

- When the bias-precession-nutation matrix is already known, the CIP $X,Y$ can be extracted by calling `iau_BPN2XY`.
- The IAU 2000A $X,Y$, together with the CIO locator $s$, can be obtained using `iau_XYS00A`.
- For the IAU 2000B model, $X,Y$ and $s$ can be obtained by calling `iau_XYS00B`.
- When the bias-precession-nutation is available in the form of Fukushima-Williams angles, the CIP coordinates can be computed with `iau_FW2XY`.
- In the case of the IAU 2006/2000A model, SOFA provides a routine `iau_XY06` that computes $X$ and $Y$ directly from series. This formulation is distinct from the other SOFA IAU 2006/2000A routines but agrees at the 1 $\mu$as level.
3.4 The CIO locator, $s$

Although direct series for $s$ exist, a more concise result is obtained if the quantity modeled is not $s$ itself but $s + XY/2$, where $X, Y$ are the CIP coordinates. SOFA provides three routines for $s$ in the IAU 2000 system...

- The routine `iau_S00A` calculates $s$, first evaluating the IAU 2000A bias-precession-nutation to obtain the CIP coordinates $X$ and $Y$.
- The routine `iau_S00B` does the same but using the IAU 2000B model.
- Where $X$ and $Y$ are already known, it is computationally more efficient to call the routine `iau_S00`.

...and two routines for $s$ in the IAU 2006/2000A system:

- The routine `iau_S06A` calculates $s$, first evaluating the IAU 2006/2000A bias-precession-nutation to obtain the CIP coordinates $X$ and $Y$.
- Where $X$ and $Y$ are already known (for example from `iau_XY06`), it is computationally more efficient to call the routine `iau_S06`.

3.5 Polar motion

The polar motion matrix, $R_{PM}$ in Eq. (1) depends on IERS tabulations of the pole coordinates $x_p, y_p$. The SOFA routine `iau_POM00` uses these pole coordinates, together with the TIO locator $s'$ to construct the $R_{PM}$ matrix. A conventional estimate of $s'$ is provided by the routine `iau_SP00`. Prior to the introduction of the IAU 2000 procedures $s'$ was neglected.

3.6 Earth rotation

In Eq. 1, the angle $\theta$ is either Greenwich apparent sidereal time (if $R_{NPB}$ is equinox based) or Earth rotation angle (if $R_{NPB}$ is CIO based). SOFA includes routines to calculate both.

Support for classical sidereal time accounts for a total of 14 SOFA routines. The large number is because:

- Sidereal time includes a precession-nutation component as well as the main Earth rotation component. Consequently, each new precession-nutation model requires its own matching sidereal time model.
Sidereal time has several component parts. The ST that is \( \theta \) in the expression:

\[
h_{\text{Greenwich}} = \theta - \alpha_{\text{apparent}}
\]

is *apparent* sidereal time and comprises:

\[
\theta = \text{GMST} + \text{EE}
\]

where EE is the *equation of the equinoxes*. The classical equation of the equinoxes is \( \Delta \psi \cos \epsilon \), where \( \Delta \psi \) is the nutation in longitude and \( \epsilon \) is the obliquity, but from 1994 small periodic corrections called the *complementary terms* have been added.

SOFA provides three generations of IAU GMST models:

- `iau_GMST82` computes the IAU 1982 GMST, consistent with the IAU 1976 precession.
- `iau_GMST00` computes the GMST consistent with the precession part of IAU 2000A.
- `iau_GMST06` computes the GMST consistent with the IAU 2006 precession.

Five SOFA routines deliver the equation of the equinoxes:

- `iau_EQEQ94` computes the IAU 1994 EE.
- `iau_EE00A` computes the IAU 2000A EE.
- `iau_EE00B` computes the IAU 2000B EE.
- Where \( \Delta \psi \) and \( \epsilon \) are already available, `iau_EE00` can be called, producing the IAU 2000 EE.
- `iau_EE06A` computes the IAU 2006/2000A EE.

The SOFA routine `iau_EECT00` computes the value of the EE complementary terms for the IAU 2000 case. No corresponding IAU 2000/2000A routine exists; should the quantity be needed, it can be computed by subtracting \( \Delta \psi \cos \epsilon \) from the EE, the latter obtained by calling `iau_EE06A`.

Finally, SOFA routines are provided for calculating five generations of Greenwich apparent sidereal time:

- `iau_GST94` computes the IAU 1994 GST.
- `iau_GST00A` computes the IAU 2000A GST.
- `iau_GST00B` computes the IAU 2000B GST.
- `iau_GST06` computes the GST starting from the NPB matrix. Although the routine uses the IAU 2006/2000A series for \( s + XY/2 \) (hence its “06” suffix), it is otherwise independent of the precession-nutation model and can in practice be used with any equinox based NPB matrix.
• iau_GST06A computes the IAU 2006/2000A GST.

Working with respect to the CIO rather than the equinox sweeps away sidereal time’s complexities and opportunities for error. The Earth rotation angle, the CIO based counterpart of GST, is simply a conventional linear transformation of UT1, and only one SOFA routine is needed:

• iau_ERA00: Earth rotation angle.

3.7 Fundamental arguments

The models for nutation, the CIO locator $s$ and the CIP $X, Y$ are all functions of fundamental arguments, angles which themselves are functions of time. SOFA routines implement the IERS 2003 expressions for 14 such angles:

- iau_FAL03 Delaunay variable $l$
- iau_FALP03 Delaunay variable $l'$
- iau_FAF03 Delaunay variable $F$
- iau_FAD03 Delaunay variable $D$
- iau_FAOM03 Delaunay variable $\Omega$
- iau_FAME03 mean longitude of Mercury
- iau_FAVE03 mean longitude of Venus
- iau_FAE03 mean longitude of Earth
- iau_FAMA03 mean longitude of Mars
- iau_FAJU03 mean longitude of Jupiter
- iau_FASA03 mean longitude of Saturn
- iau_FAUR03 mean longitude of Uranus
- iau_FANE03 mean longitude of Neptune
- iau_FAPA03 general accumulated precession in longitude

In a few cases the published models use slightly different expressions for one or two of the fundamental arguments, and where this occurs the SOFA routine uses the published expression rather than the standard version.

3.8 Supporting routines

The successive generations of precession-nutation models are supported by SOFA routines that generate the various matrices from a variety of starting points.

The IAU 1976/1980 models are used by iau_PNM80.

The IAU 2000A/B models are used by iau_C2I00A, iau_C2I00B, iau_C2IBPN, iau_C2T00A, iau_C2T00B, iau_C2TPE, iau_C2TXY, iau_PN00, iau_PN00A, iau_PN00B, iau_PNM00A and iau_PNM00B.

SOFA support for the current generation of models, based on the IAU 2006/2000A precession-nutation, is described in more detail in the next Section.
4 CURRENT MODELS

The latest models supported by SOFA are based on the IAU 2006 precession, the IERS 2003 polar motion and the IAU 2000 nutation and Earth rotation. With one exception (iau_XY06), all the SOFA routines rest upon a non-redundant set of models, guaranteeing self consistency: the same quantity computed via different sequences of SOFA calls should not vary apart from rounding errors.

4.1 Canonical basis

The models that form the primary canonical basis for the latest SOFA routines, supporting both CIO based and equinox based applications, are as follows:

- The IAU 2006 precession model, in the Fukushima-Williams four angle parameterization.
- The IAU 2000A nutation model.
- The IAU 2006 adjustments to the IAU 2000A nutation model.
- The IAU 2006 model for the quantity \( s + XY/2 \), which locates the celestial intermediate origin on the CIP equator.
- The IAU 2000 model for the quantity \( s' \), which locates the terrestrial intermediate origin on the CIP equator.
- The IAU 2000 expression for ERA(UT1).
- A GMST model consistent with IAU 2006.

The derived products are:

- Precession-nutation matrices, for transforming between GCRS and apparent place (either CIO based or equinox based).
- CIP coordinates \( X, Y \).
- Equation of the origins, \( EO = ERA - GST \).
- Greenwich (apparent) sidereal time, \( GST \).
- Equation of the equinoxes, \( EE = GST - GMST \).

An alternative canonical basis for the CIO based products is:

- Direct series for the IAU 2006/2000A CIP \( X, Y \).
- The IAU 2006/2000A model for \( s + XY/2 \).
• The IAU 2000 model for $s'$.
• The IAU 2000 expression for ERA(UT1).

To support equinox based applications the following additional models are required:

• The first two Fukushima-Williams angles, namely $\bar{\gamma}$ and $\bar{\phi}$.
• A GMST model consistent with IAU 2006.

The angles based and series based approaches agree to of order 1 $\mu$as; anyone attempting to mix the two techniques in a single application should not be surprised when discrepancies of this size appear. See Wallace, P.T. & Capitaine, N., Precession-nutation procedures consistent with IAU 2006 resolutions, Astronomy & Astrophysics, 459, 981-985, 2006 for further details of the two methods.

4.2 SOFA routines

With the exception of iau_XY06, which computes CIP $X, Y$ directly by series, and iau_P06E, which calculates all the equinox based IAU 2006 precession angles, all of the SOFA routines whose names contain “06” stem from the primary canonical basis set out at the start of Section 4. Hence the remainder of the present Section applies to the primary canonical basis rather than the alternative $X, Y$ series starting point. Also, with the partial exception of the low-level routines iau_PN06, iau_PN06A and iau_BP06, which keep frame bias and precession separate, everything refers to the GCRS pole rather than the dynamical pole.

For any given date, the complete matrix $R_{NPB}$ in Eq. (1) can be obtained by calling the routine iau_C2I06A, for the CIO based version, or iau_PNM06A, for the equinox based version.

At a lower level, the routine iau_PFW06 returns the Fukushima-Williams precession angles themselves (with frame bias included), while iau_NUT06A obtains the nutation components. Once these nutation components have been added to the appropriate two precession angles, the equinox based $R_{NPB}$ matrix can be generated by calling iau_Fw2M. Alternatively, the CIP $X, Y$ coordinates can be obtained by calling iau_Fw2XY, the CIO locator $s$ computed using the routine iau_S06A, and the CIO based $R_{NPB}$ matrix formed using iau_C2IXYS. This latter route provides an opportunity to introduce corrections to $X, Y$, as published by the IERS. The resulting matrix transforms from GCRS to CIRS; its transpose, available by calling iau_TR, performs the inverse transformation, i.e. CIRS to GCRS.

To support older applications that use the Euler angles $\zeta, z, \theta$ to construct the precession matrix, the routine iau_PB06 is provided. The starting point is the Fukushima-Williams angles, so that the $\zeta, z, \theta$ returned by iau_PB06 are consistent with the primary canonical basis, and include frame bias.

Earth rotation angle, from which $R_3(\theta)$ in the CIO based form of Expression (1) can be applied, is available using iau_ERA00, which is a function of UT1 alone. Or, for the case where the
equinox based form of Expression (1) is being used, the requisite Greenwich apparent sidereal
time can obtained by calling \texttt{iau\_GST06A}, a function of time \textit{(i.e. TT)} as well as UT1.

A rotation matrix that performs the complete celestial to terrestrial transformation (\textit{i.e.} GCRS
to ITRS), given the TT, UT1 and polar motion, can be obtained by calling \texttt{iau\_C2T06A}. Low-
level routines \texttt{iau\_C2TCIO} and \texttt{iau\_C2TEQX} (which are in fact identical in form) construct the
GCRS to ITRS matrix given an appropriate NPB matrix, a polar motion matrix, and the ERA
or GST.

A key component of the GST calculation is the equation of the origins, which can be obtained
as a function of time by calling \texttt{iau\_EO06A}. Alternatively, to obtain the EO corresponding to
an independently calculated \textbf{R} \textsubscript{NPB} matrix and CIO locator, \texttt{iau\_EORS} can be called. As well
as its role in computing GST, the EO is useful when CIRS right ascension is to computed from
apparent right ascension or \textit{vice versa}. Should the application require GMST or EE, the former
can be computed by calling \texttt{iau\_GMST06} and the latter by subtracting GMST from GST.

Section 5.5 demonstrates how to generate the CIO based celestial to terrestrial transformation
with the usual angles based method. Section 5.6 does the same but starting from the \textit{X}, \textit{Y} series.
In both cases IERS CIP corrections and Earth orientation parameters are taken into account.

5 EXAMPLES

Sections 5.1–5.6, below, present a complete Fortran program that computes the GCRS-to-ITRS
transformation matrix and various intermediate products, using a number of different approaches
and generations of SOFA routines. The code is interspersed with explanatory text and numerical
results, but is otherwise compilable as it stands.

Five methods are demonstrated:

5.3 IAU 2000A, CIO based, using classical angles
5.4 IAU 2000A, equinox based, using classical angles
5.5 IAU 2006/2000A, CIO based, using classical angles
5.6 IAU 2006/2000A, CIO based, using \textit{X}, \textit{Y} series

The starting point is as follows:

\begin{align*}
\text{date} &\quad \text{time} &\quad 2007 \text{ April} 05, \ 12^h \ 00^m \ 00^s \ \text{UTC} \\
x_p &\quad +0^\circ 0349282 \\
y_p &\quad +0^\circ 4833163 \\
\text{UT1-UTC} &\quad -0^\circ 072073685 \\
d\psi_{1980} &\quad -0^\circ 0550655 \\
d\epsilon_{1980} &\quad -0^\circ 0063580
\end{align*}
\[
\begin{align*}
\Delta X_{2000} &= +0.0001725 \\
\Delta Y_{2000} &= -0.0002650 \\
\Delta X_{2006} &= +0.0001750 \\
\Delta Y_{2006} &= -0.0002259 
\end{align*}
\]

The Earth orientation parameters, which are available from the IERS, are corrections to the standard Earth models and come from observation: the polar motion quantities \( x_p \) and \( y_p \) correct the terrestrial pole, UT1–UTC is the time correction that defines the Earth rotation, and \( \Delta X_{2000} \) and \( \Delta Y_{2000} \) are corrections to the celestial pole with respect to the IAU 2000A model. (n.b. The above values are purely for illustration but in fact are fairly close to those for the given date.)

The quantities \( \Delta X_{2006} \) and \( \Delta Y_{2006} \) are adjusted versions of \( \Delta X_{2000} \) and \( \Delta Y_{2000} \) that for the purposes of this demonstration correct the celestial pole with respect to the IAU 2006/2000A model.\(^2\)

### 5.1 Preliminaries

The program begins as follows:

```fortran
* SOFA examples
IMPLICIT NONE

* Arcseconds to radians
DOUBLE PRECISION AS2R
PARAMETER ( AS2R = 4.848136811095359935899141D-6 )

INTEGER IY, IM, ID, IH, MIN, J
DOUBLE PRECISION SEC, XP, YP, DUT1,
: DDP80, DDE80, DX00, DY00, DX06, DY06,
: UTC1, UTC2, TAI1, TAI2, TT1, TT2, UT11, UT12,
: RP(3,3), DP80, DE80,
: DPSI, DEPS, EPSA, RN(3,3), RNPB(3,3),
: EE, GST, RC2TI(3,3), RPM(3,3),
: RC2IT(3,3), X, Y, S,
: RC2I(3,3), ERA, SP, DP00, DE00, RB(3,3),
: RPB(3,3), V1(3), V2(3), DDP00, DDE00
DOUBLE PRECISION iau_OBL80, iau_EQEQ94, iau_ANP, iau_GMST82,
: iau_ERA00, iau_SPO0, iau_EEO0, iau_GMST00,
: iau_S06
```

\(^2\)The adjustments were computed by calling \texttt{iau\_XYS00A} and \texttt{iau\_XYS06A} for the given date and subtracting the respective X,Y coordinates.

* UTC.
  IY = 2007
  IM = 4
  ID = 5
  IH = 12
  MIN = 0
  SEC = 0D0

* Polar motion (arcsec->radians).
  XP = 0.0349282D0 * AS2R
  YP = 0.4833163D0 * AS2R

* UT1-UTC (s).
  DUT1 = -0.072073685D0

  DDP80 = -55.0655D0 * AS2R/1000D0
  DDE80 = -6.3580D0 * AS2R/1000D0

* CIP offsets wrt IAU 2000A (mas->radians).
  DX00 = 0.1725D0 * AS2R/1000D0
  DY00 = -0.2650D0 * AS2R/1000D0

* CIP offsets wrt IAU 2006/2000A (mas->radians).
  DX06 = 0.1750D0 * AS2R/1000D0
  DY06 = -0.2259D0 * AS2R/1000D0

* From UTC get TT and UT1.
  CALL iau_DTF2D ( 'UTC', IY, IM, ID, IH, MIN, SEC, UTC1, UTC2, J )
  CALL iau_UTCTAI ( UTC1, UTC2, TAI1, TAI2, J )
  CALL iau_TAITT ( TAI1, TAI2, TT1, TT2, J )
  CALL iau_UTCUT1 ( UTC1, UTC2, DUT1, UT11, UT12, J )

The resulting TT and UT1 are as follows:

\[
TT = 2454195.5 + 0.500754444444444
\]
\[
UT1 = 2454195.5 + 0.499999165813831
\]

n.b. Here and elsewhere the numerical results are given to an unrealistic precision, for comparison purposes.


The next part of the Fortran code demonstrates the procedures that were used prior to the introduction (at the beginning of 2003) of the methods specified in the IAU 2000 resolutions.
* =============
* IAU 1976/1980
* =============

* IAU 1976 precession matrix, J2000.0 to date.
   CALL iau_PMAT76 ( TT1, TT2, RP )

* IAU 1980 nutation.
   CALL iau_NUT80 ( TT1, TT2, DP80, DE80 )

* Add adjustments: frame bias, precession-rates, geophysical.
   DPSI = DP80 + DDP80
   DEPS = DE80 + DDE80

* Mean obliquity.
   EPSA = iau_OBL80 ( TT1, TT2 )

* Build the rotation matrix.
   CALL iau_NUMAT ( EPSA, DPSI, DEPS, RN )

* Combine the matrices: PN = N x P.
   CALL iau_RXR ( RN, RP, RNPB )

* Equation of the equinoxes, including nutation correction.
   EE = iau_EQEQ94 ( TT1, TT2 ) + DDP80 * COS(EPSA)

   GST = iau_ANP ( iau_GMST82 ( UT11, UT12 ) + EE )

* Form celestial-terrestrial matrix (no polar motion yet).
   CALL iau_CR ( RNPB, RC2TI )
   CALL iau_RZ ( GST, RC2TI )

   CALL iau_IR ( RPOM )
   CALL iau_RX ( -YP, RPOM )
   CALL iau_RY ( -XP, RPOM )

* Form celestial-terrestrial matrix (including polar motion).
   CALL iau_RXR ( RPOM, RC2TI, RC2IT )

The results are as follows:

NPB matrix, equinox based
At the start of 2003, the IAU 2000A precession-nutation model came into use. The following code uses classical precession and nutation angles to obtain the CIP coordinates \(X, Y\), then uses the CIO locator \(s\) and the Earth rotation angle.

\[
\begin{pmatrix}
+0.999998403176203 & -0.0016390329720562 & -0.000712190961847 \\
+0.0016390094224243 & +0.99998655799521 & -0.000045528466242 \\
+0.000712264667137 & +0.00044385492226 & +0.999997453545454
\end{pmatrix}
\]

\[
\text{GST} = 13\degree 41\text{m} 23\text{s} 976571378
\]

\[
\begin{pmatrix}
+0.973104317592265 & +0.230363826166883 & -0.000703332813776 \\
-0.230363798723533 & +0.973104570754697 & +0.000120888299841 \\
+0.000712264667137 & +0.00044385492226 & +0.999997453545454
\end{pmatrix}
\]

\[
\begin{pmatrix}
+0.973104317712772 & +0.230363826174782 & -0.000703163477127 \\
-0.230363800391868 & +0.973104570648022 & +0.000118545116892 \\
+0.000711560100206 & +0.00046626645796 & +0.99999745754058
\end{pmatrix}
\]
   CALL iau_POM00 ( XP, YP, iau_SP00(TT1,TT2), RPOM )

* Form celestial-terrestrial matrix (including polar motion).
   CALL iau_RXR ( RPOM, RC2TI, RC2IT )

The results are as follows:

\[
\begin{align*}
X &= +0.000712264729708 \\
Y &= +0.000044385250265 \\
s &= -0'002200496
\end{align*}
\]

NPB matrix, CIO based

\[
\begin{pmatrix}
+0.999999746339445 & -0.00000005138721 & -0.000712264730182 \\
-0.00000026475329 & +0.99999999014975 & -0.000044385242666 \\
+0.000712264729708 & +0.000044385250265 & +0.999999745354420
\end{pmatrix}
\]

\[
\begin{align*}
ERA &= 13\text{h}31\text{m}49\text{s}66097 \\
     &= 00^h53^m16^s438311863
\end{align*}
\]

Celestial to terrestrial matrix (no polar motion)

\[
\begin{pmatrix}
+0.973104317573104 & +0.230363826247808 & -0.000703332818915 \\
-0.230363798804281 & +0.973104570735550 & +0.000120888549767 \\
+0.000712264729708 & +0.000044385250265 & +0.999999745354420
\end{pmatrix}
\]

Celestial to terrestrial matrix

\[
\begin{pmatrix}
+0.973104317697512 & +0.230363826239227 & -0.000703163482268 \\
-0.230363800456136 & +0.973104570632777 & +0.000118545366806 \\
+0.000711560162777 & +0.000046626403835 & +0.999999745754024
\end{pmatrix}
\]

The rotational difference between the final matrices of this and the previous method is 53.29 $\mu$as.

5.4 IAU 2000A, equinox based, using classical angles

Equinox based applications could instead do the following. In this case the nutation corrections ($d\psi, d\epsilon$) must be obtained by appropriately transforming the CIP corrections ($dX, dY$). It is necessary to select the GMST and EE routines that match the IAU 2000A precession, in this case iau_GMST00 and iau_EE00.
5.4 IAU 2000A, equinox based, using classical angles

* ========================
* IAU 2000A, equinox based
* ========================

* Nutation, IAU 2000A.
  CALL iau_NUT00A ( TT1, TT2, DP00, DE00 )

  CALL iau_PN00 ( TT1, TT2, DP00, DE00,
  : EPSA, RB, RP, RPB, RN, RNPB )

* Transform dX,dY corrections from GCRS to mean of date.
  V1(1) = DX00
  V1(2) = DY00
  V1(3) = 0D0
  CALL iau_RXP ( RNPB, V1, V2 )
  DDP00 = V2(1) / SIN(EPSA)
  DDE00 = V2(2)

* Corrected nutation.
  DPSI = DP00 + DDP00
  DEPS = DE00 + DDE00

* Build the rotation matrix.
  CALL iau_NUMAT ( EPSA, DPSI, DEPS, RN )

* Combine the matrices: PN = N x P.
  CALL iau_RXR ( RN, RPB, RNPB )

* Greenwich apparent sidereal time (IAU 2000).
  GST = iau_ANP ( iau_GMST00 ( UT11, UT12, TT1, TT2 )
  : + iau_EE00 ( TT1, TT2, EPSA, DPSI ) )

* Form celestial-terrestrial matrix (no polar motion yet).
  CALL iau_CR ( RNPB, RC2TI )
  CALL iau_RZ ( GST, RC2TI )

  SP = iau_SP00 ( TT1, TT2 )
  CALL iau_POM00 ( XP, YP, SP, RPOM )

* Form celestial-terrestrial matrix (including polar motion).
  CALL iau_RXR ( RPOM, RC2TI, RC2IT )
The results are as follows:

NPB matrix, equinox based

\[
\begin{pmatrix}
+0.999998402755640 & -0.001639289519579 & -0.000712191013215 \\
+0.001639257491365 & +0.999998655379006 & -0.000044385250265 \\
+0.000712264729795 & +0.00044385250265 & +0.999999745354420
\end{pmatrix}
\]

GST = 13°41'24.17084674 = 00h 53m 38s .980100322

celestial to terrestrial matrix (no polar motion)

\[
\begin{pmatrix}
+0.973104317573209 & +0.230363826247361 & -0.000703332818999 \\
-0.230363798803834 & +0.973104570735656 & +0.000120888549787 \\
+0.000712264729795 & +0.00044385250265 & +0.999999745354420
\end{pmatrix}
\]

celestial to terrestrial matrix

\[
\begin{pmatrix}
+0.973104317697618 & +0.230363826238780 & -0.000703163482352 \\
-0.230363800455689 & +0.973104570632883 & +0.000118545366826 \\
+0.000711560162864 & +0.00046626403835 & +0.999999745754024
\end{pmatrix}
\]

The rotational difference between the final matrices of this and the previous method (5.3) is 0.10 µas.

5.5 IAU 2006/2000A, CIO based, using classical angles

From the beginning of 2009 the IAU 2006 precession will be used. The procedure based on classical precession and mutation angles is essentially the same as before (see Section 5.3), the only change being to call iau_XYS06A instead of iau_XYS00A.

* IAU 2006/2000A, CIO based
* CIP and CIO, IAU 2006/2000A.
  CALL iau_XYS06A ( TT1, TT2, X, Y, S )
* Add CIP corrections.
  X = X + DX06
  Y = Y + DY06
* GCRS to CIRS matrix.
  CALL iau_C2IXYS ( X, Y, S, RC2I )
* Earth rotation angle.
  ERA = iau_ERA00 ( UT11, UT12 )

* Form celestial-terrestrial matrix (no polar motion yet).
  CALL iau_CR ( RC2I, RC2TI )
  CALL iau_RZ ( ERA, RC2TI )

  SP = iau_SP00 ( TT1, TT2 )
  CALL iau_POM00 ( XP, YP, SP, RPOM )

* Form celestial-terrestrial matrix (including polar motion).
  CALL iau_RXR ( RPOM, RC2TI, RC2IT )

The results are as follows:

\[
\begin{pmatrix}
+0.000712264729599 \\
+0.000044385250426 \\
-0.002200475
\end{pmatrix}
\]

NPB matrix, CIO based

\[
\begin{pmatrix}
+0.9999999746339445 & -0.00000005138822 & -0.000712264730072 \\
-0.00000026475227 & +0.99999999014975 & -0.000044385242827 \\
+0.000712264729599 & +0.000044385250426 & +0.999999745354420
\end{pmatrix}
\]

ERA = 13.318492966097 = 00\textdegree 53' 16'' 43.8311863

celestial to terrestrial matrix (no polar motion)

\[
\begin{pmatrix}
+0.973104317573127 & +0.230363826247709 & -0.000703332818845 \\
-0.230363798804182 & +0.973104570735574 & +0.000120888549586 \\
+0.000712264729599 & +0.000044385250426 & +0.999999745354420
\end{pmatrix}
\]

celestial to terrestrial matrix

\[
\begin{pmatrix}
+0.973104317697535 & +0.230363826239128 & -0.000703163482198 \\
-0.23036380045637 & +0.973104570632801 & +0.00011854366625 \\
+0.000711560162668 & +0.000046626403995 & +0.999999745754024
\end{pmatrix}
\]

The rotational difference between the final matrices of this and method (5.3) is 0.05\textmu as.
* IAU 2006/2000A, CIO based, using X,Y series

* CIP and CIO, IAU 2006/2000A.
  CALL iau_XY06 ( TT1, TT2, X, Y )
  S = iau_S06 ( TT1, TT2, X, Y )

* Add CIP corrections.
  X = X + DX06
  Y = Y + DY06

* GCRS to CIRS matrix.
  CALL iau_C2IXYS ( X, Y, S, RC2I )

* Earth rotation angle.
  ERA = iau_ERA00 ( UT11, UT12 )

* Form celestial-terrestrial matrix (no polar motion yet).
  CALL iau_CR ( RC2I, RC2TI )
  CALL iau_RZ ( ERA, RC2TI )

  SP = iau_SP00 ( TT1, TT2 )
  CALL iau_POM00 ( XP, YP, SP, RPOM )

* Form celestial-terrestrial matrix (including polar motion).
  CALL iau_RXR ( RPOM, RC2TI, RC2IT )

END

The results are as follows:

X = +0.000712264729525
Y = +0.000044385248875
s = −0.002200475

NPB matrix, CIO based

\[
\begin{pmatrix}
+0.999999746339445 & -0.00000005138822 & -0.000712264729999 \\
-0.000000026475226 & +0.99999999014975 & -0.000044385241276 \\
+0.000712264729525 & +0.000044385248875 & +0.999999745354420
\end{pmatrix}
\]

ERA = 13°318492966097
= 00h 53m 16.438311863

celestial to terrestrial matrix (no polar motion)

\[
\begin{pmatrix}
+0.973104317573127 & +0.230363826247709 & -0.00070332818416 \\
-0.230363798804181 & +0.973104570735574 & +0.000120888551078 \\
+0.000712264729525 & +0.00044385248875 & +0.999999745354420
\end{pmatrix}
\]

celestial to terrestrial matrix

\[
\begin{pmatrix}
+0.973104317697536 & +0.230363826239128 & -0.000703163481769 \\
-0.230363800456036 & +0.973104570632801 & +0.000118545368117 \\
+0.000711560162594 & +0.00046626402444 & +0.999999745754024
\end{pmatrix}
\]

The rotational difference between the final matrices of this and method (5.3) is 0.29 μas.

6 FURTHER READING

For numerical examples, with explanatory material, see The Astronomical Almanac and similar national publications. An extensive glossary is available as part of The Astronomical Almanac Online: see http://asa.hmnao.com/SecM/Glossary.html.

Detailed information on models and procedures can be found in:


- The celestial pole coordinates, N. Capitaine, Celestial Mechanics & Dynamical Astronomy, 48, 2, 127-143 (1990)


The following may also be useful for background reading:


- Vectorial Astrometry, C. A. Murray (1983), Adam Hilger

- Spherical Astronomy, Robin M. Green (1985), Cambridge University Press

- Spacecraft Attitude Determination and Control, ed. James R. Wertz (1986), Reidel

7 ROUTINE SPECIFICATIONS

The following pages present in alphabetical order details of all the SOFA routines referred to in the present document. These are mainly concerned with the Earth’s orientation and rotation, but for convenience a few other routines that happen to be used in the code examples are included as well.

\textbf{iau\_ANP} \hspace{1cm} \textit{normalize radians to range 0 to }2\pi \hspace{1cm} \textbf{iau\_ANP}

\textsc{call}:
\begin{verbatim}
  D = iau\_ANP ( A )
\end{verbatim}

\textsc{action}:
Normalize angle into the range \(0 \leq A < 2\pi\).

\textsc{given}:
\begin{tabular}{l}
  \(A\) \hspace{1cm} \text{d} \hspace{1cm} \text{angle (radians)}
\end{tabular}

\textsc{returned} (function value):
\begin{tabular}{l}
  \text{d} \hspace{1cm} \text{angle in range 0-2\pi}
\end{tabular}
CALL :  
CALL iau_BI00 ( DPSIBI, DEPSBI, DRA )

ACTION :  
Frame bias components of IAU 2000 precession-nutation models; part of the Mathews-Herring-Buffett (MHB2000) nutation series, with additions.

RETURNED :  

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DPSIBI</td>
<td>longitude correction (radians)</td>
</tr>
<tr>
<td>DEPSBI</td>
<td>obliquity correction (radians)</td>
</tr>
<tr>
<td>DRA</td>
<td>the ICRS RA of the J2000.0 mean equinox (radians)</td>
</tr>
</tbody>
</table>

NOTES :  
1. The frame bias corrections in longitude and obliquity (radians) are required in order to correct for the offset between the GCRS pole and the J2000.0 mean pole. They define, with respect to the GCRS frame, a J2000.0 mean pole that is consistent with the rest of the IAU 2000A precession-nutation model.

2. In addition to the displacement of the pole, the complete description of the frame bias requires also an offset in right ascension. This is not part of the IAU 2000A model, and is from Chapront et al. (2002). It is returned in radians.

3. This is a supplemented implementation of one aspect of the IAU 2000A nutation model, formally adopted by the IAU General Assembly in 2000, namely MHB2000 (Mathews et al. 2002).

REFERENCES :  
**CALL:**
CALL iau_BPN2XY ( RBPN, X, Y )

**ACTION:**
Extract from the bias-precession-nutation matrix the $X,Y$ coordinates of the Celestial Intermediate Pole.

**GIVEN:**
$RBPN$  \text{d}(3,3) \quad \text{celestial-to-true matrix (Note 1)}

**RETURNED:**
$X,Y$  \text{d}  \quad \text{Celestial Intermediate Pole (Note 2)}

**NOTES:**
1. The matrix $RBPN$ transforms vectors from GCRS to true equator (and CIO or equinox) of date, and therefore the Celestial Intermediate Pole unit vector is the bottom row of the matrix.
2. $X,Y$ are components of the Celestial Intermediate Pole unit vector in the Geocentric Celestial Reference System.

**REFERENCE:**
CALL:

CALL iau_C2I00A ( DATE1, DATE2, RC2I )

ACTION:

Form the celestial-to-intermediate matrix for a given date using the IAU 2000A precession-nutation model.

GIVEN:

DATE1  d  TT as a two-part...
DATE2  d  ...Julian Date (Note 1)

RETURNED:

RC2I  d(3,3)  celestial-to-intermediate matrix (Note 2)

NOTES:

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

   DATE1    DATE2
   2450123.7D0  0D0  (JD method)
   2451545.0D0 −1421.3D0  (J2000 method)
   2400000.5D0  50123.2D0  (MJD method)
   2450123.5D0  0.2D0  (date & time method)

   The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The matrix RC2I is the first stage in the transformation from celestial to terrestrial coordinates:

   \[ \mathbf{v}_{\text{TRS}} = \mathbf{R}_{\text{POM}} \mathbf{R}_3(\text{ERA}) \mathbf{R}_{\text{C2I}} \mathbf{v}_{\text{CRS}} \equiv \mathbf{R}_{\text{C2T}} \mathbf{v}_{\text{CRS}} \]

   where \( \mathbf{v}_{\text{TRS}} \) is a vector in the International Terrestrial Reference System (see IERS Conventions 2003), \( \mathbf{R}_{\text{POM}} \) is the polar motion matrix, \( \text{ERA} \) is the Earth Rotation Angle, \( \mathbf{R}_{\text{C2I}} \) is the matrix RC2I returned by the present routine, and \( \mathbf{v}_{\text{CRS}} \) is a vector in the Geocentric Celestial Reference System.

3. A faster, but slightly less accurate, result (about 1 mas) can be obtained by using instead the iau_C2I00B routine.
REFERENCES:


CALL :  
CALL iau_C2I00B ( DATE1, DATE2, RC2I )

ACTION :  
Form the celestial-to-intermediate matrix for a given date using the IAU 2000B precession-nutation model.

GIVEN :  
\[
\begin{align*}
\text{DATE1} & \quad \text{d} \quad \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} \quad \ldots \text{Julian Date (Note 1)}
\end{align*}
\]

RETURNED :  
\[
\begin{align*}
\text{RC2I} & \quad \text{d(3,3)} \quad \text{celestial-to-intermediate matrix (Note 2)}
\end{align*}
\]

NOTES :  
1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

\[
\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 \quad \text{(JD method)} \\
2451545.0D0 & \quad -1421.3D0 \quad \text{(J2000 method)} \\
2400000.5D0 & \quad 50123.2D0 \quad \text{(MJD method)} \\
2450123.5D0 & \quad 0.2D0 \quad \text{(date & time method)}
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The matrix RC2I is the first stage in the transformation from celestial to terrestrial coordinates:

\[
\begin{align*}
\mathbf{v}_{\text{TRS}} &= \mathbf{R}_{\text{POM}} \mathbf{R}_3(ERA) \mathbf{R}_{\text{C2I}} \mathbf{v}_{\text{CRS}} \\
&\equiv \mathbf{R}_{\text{C2T}} \mathbf{v}_{\text{CRS}}
\end{align*}
\]

where \( \mathbf{v}_{\text{TRS}} \) is a vector in the International Terrestrial Reference System (see IERS Conventions 2003), \( \mathbf{R}_{\text{POM}} \) is the polar motion matrix, \( ERA \) is the Earth Rotation Angle, \( \mathbf{R}_{\text{C2I}} \) is the matrix RC2I returned by the present routine, and \( \mathbf{v}_{\text{CRS}} \) is a vector in the Geocentric Celestial Reference System.

3. The present routine is faster, but slightly less accurate (about 1 mas), than the iau_C2I00A routine.
REFERENCES:


CALL :  
CALL iau_C2IBPN ( DATE1, DATE2, RBPN, RC2I )

ACTION :  
Form the celestial-to-intermediate matrix for a given date given the bias-precession-nutation matrix. IAU 2000.

GIVEN :  
DATE1 d TT as a two-part...
DATE2 d ...Julian Date (Note 1)
RBPN d(3,3) celestial-to-true matrix (Note 2)

RETURNED :  
RC2I d(3,3) celestial-to-intermediate matrix (Note 3)

NOTES :  
1. The TT date $DATE1 + DATE2$ is a Julian Date, apportioned in any convenient way between the arguments $DATE1$ and $DATE2$. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

<table>
<thead>
<tr>
<th>$DATE1$</th>
<th>$DATE2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2450123.7D0</td>
<td>0D0</td>
</tr>
<tr>
<td>2451545.0D0</td>
<td>1421.3D0</td>
</tr>
<tr>
<td>2400000.5D0</td>
<td>50123.2D0</td>
</tr>
<tr>
<td>2450123.5D0</td>
<td>0.2D0</td>
</tr>
</tbody>
</table>

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The matrix $RBPN$ transforms vectors from GCRS to true equator (and CIO or equinox) of date. Only the CIP (bottom row) is used.

3. The matrix $RC2I$ is the first stage in the transformation from celestial to terrestrial coordinates:

$$v_{TRS} = R_{POM} R_{3}(ERA) R_{C2I} v_{CRS}$$

$$\equiv R_{C2T} v_{CRS}$$

where $v_{TRS}$ is a vector in the International Terrestrial Reference System (see IERS Conventions 2003), $R_{POM}$ is the polar motion matrix, $ERA$ is the Earth Rotation Angle, $R_{C2I}$ is the matrix $RC2I$ returned by the present routine, and $v_{CRS}$ is a vector in the Geocentric Celestial Reference System.

4. Although its name does not include “00”, this routine is in fact specific to the IAU 2000 models.
REFERENCES:


CALL:

CALL iau_C2T00A ( TTA, TTB, UTA, UTB, XP, YP, RC2T )

ACTION:

Form the celestial to terrestrial matrix given the date, the UT1 and the polar motion, using the IAU 2000A precession-nutation model.

GIVEN:

| TTA, TTB   | d   | TT as a two-part Julian Date (Note 1) |
| UTA, UTB   | d   | UT1 as a two-part Julian Date (Note 1) |
| XP, YP     | d   | coordinates of the pole (radians, Note 2) |

RETURNED:

| RC2T       | d(3,3) | celestial-to-terrestrial matrix (Note 3) |

NOTES:

1. The TT and UT1 dates TTA+TTB and UTA+UTB are Julian Dates, apportioned in any convenient way between the arguments UTA and UTB. For example, JD(UT1) = 2450123.7 could be expressed in any of these ways, among others:

<table>
<thead>
<tr>
<th>UTA</th>
<th>UTB</th>
</tr>
</thead>
<tbody>
<tr>
<td>2450123.7D0</td>
<td>0D0</td>
</tr>
<tr>
<td>2451545.0D0</td>
<td>-1421.3D0</td>
</tr>
<tr>
<td>2400000.5D0</td>
<td>50123.2D0</td>
</tr>
<tr>
<td>2450123.5D0</td>
<td>0.2D0</td>
</tr>
</tbody>
</table>

   The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 and MJD methods are good compromises between resolution and convenience. In the case of UTA, UTB, the date & time method is best matched to the Earth rotation angle algorithm used: maximum accuracy (or, at least, minimum noise) is delivered when the UTA argument is for 0h UT1 on the day in question and the UTB argument lies in the range 0 to 1, or vice versa.

2. XP and YP are the coordinates (in radians) of the Celestial Intermediate Pole with respect to the International Terrestrial Reference System (see IERS Conventions 2003), measured along the meridians 0° and 90° west respectively.

3. The matrix RC2T transforms from celestial to terrestrial coordinates:

\[ \mathbf{v}_{\text{TRS}} = \mathbf{R}_{\text{POM}} \mathbf{R}_3(\text{ERA}) \mathbf{R}_{\text{C2I}} \mathbf{v}_{\text{CRS}} = \mathbf{R}_{\text{C2T}} \mathbf{v}_{\text{CRS}} \]

where \( \mathbf{v}_{\text{TRS}} \) is a vector in the International Terrestrial Reference System (see IERS Conventions 2003), \( \mathbf{R}_{\text{POM}} \) is the polar motion matrix, \( \text{ERA} \) is the Earth Rotation Angle, \( \mathbf{R}_{\text{C2I}} \) is the CIO based bias-precession-nutation matrix, \( \mathbf{v}_{\text{CRS}} \) is a vector in the Geocentric Celestial Reference System, and \( \mathbf{R}_{\text{C2T}} \) is the \( \text{RC2T} \) matrix returned by the present routine.
4. A faster, but slightly less accurate, result (about 1 mas) can be obtained by using instead the \texttt{iau\_C2T00B} routine.

\textbf{REFERENCE}:

CALL:

CALL iau_C2T00B ( TTA, TTB, UTA, UTB, XP, YP, RC2T )

ACTION:

Form the celestial to terrestrial matrix given the date, the UT1 and the polar motion, using the IAU 2000B precession-nutation model.

GIVEN:

TTA,TTB d TT as a two-part Julian Date (Note 1)
UTA,UTB d UT1 as a two-part Julian Date (Note 1)
XP,YP d coordinates of the pole (radians, Note 2)

RETURNED:

RC2T d(3,3) celestial-to-terrestrial matrix (Note 3)

NOTES:

1. The TT and UT1 dates TTA+TTB and UTA+UTB are Julian Dates, apportioned in any convenient way between the arguments UTA and UTB. For example,
   JD(UT1) = 2450123.7 could be expressed in any of these ways, among others:

   | UTA            | UTB            |
   |----------------|--|---------|
   | 2450123.7D0    | 0D0           | (JD method) |
   | 2451545.0D0    | -1421.3D0     | (J2000 method) |
   | 2400000.5D0    | 50123.2D0     | (MJD method) |
   | 2450123.5D0    | 0.2D0         | (date & time method) |

   The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 and MJD methods are good compromises between resolution and convenience. In the case of UTA,UTB, the date & time method is best matched to the Earth rotation angle algorithm used: maximum accuracy (or, at least, minimum noise) is delivered when the UTA argument is for 0th UT1 on the day in question and the UTB argument lies in the range 0 to 1, or vice versa.

2. XP and YP are the coordinates (in radians) of the Celestial Intermediate Pole with respect to the International Terrestrial Reference System (see IERS Conventions 2003), measured along the meridians 0° and 90° west respectively.

3. The matrix RC2T transforms from celestial to terrestrial coordinates:

\[ \mathbf{v}_{\text{TRS}} = \mathbf{R}_{\text{POM}} \mathbf{R}_3(ERA) \mathbf{R}_{\text{C2I}} \mathbf{v}_{\text{CRS}} \]

\[ \equiv \mathbf{R}_{\text{C2T}} \mathbf{v}_{\text{CRS}} \]

where \( \mathbf{v}_{\text{TRS}} \) is a vector in the International Terrestrial Reference System (see IERS Conventions 2003), \( \mathbf{R}_{\text{POM}} \) is the polar motion matrix, \( ERA \) is the Earth Rotation Angle, \( \mathbf{R}_{\text{C2I}} \) is the CIO based bias-precession-nutation matrix, \( \mathbf{v}_{\text{CRS}} \) is a vector in the Geocentric Celestial Reference System, and \( \mathbf{R}_{\text{C2T}} \) is the RC2T matrix returned by the present routine.
4. The present routine is faster, but slightly less accurate (about 1 mas), than the 
\texttt{iau\_C2T00A} routine.

\textbf{REFERENCE:}

CALL:

CALL iau_C2TEQX ( RBPN, GST, RPOM, RC2T )

ACTION:

Assemble the celestial to terrestrial matrix from equinox based components, namely the celestial-to-true matrix, the Greenwich Apparent Sidereal Time and the polar motion matrix.

GIVEN:

- RBPN $d(3,3)$ celestial-to-true matrix
- GST $d$ Greenwich (apparent) Sidereal Time (radians)
- RPOM $d(3,3)$ polar-motion matrix

RETURNED:

- RC2T $d(3,3)$ celestial-to-terrestrial matrix (Note 2)

NOTES:

1. This routine constructs the rotation matrix that transforms vectors in the celestial system into vectors in the terrestrial system. It does so starting from precomputed components, namely the matrix which rotates from celestial coordinates to the true equator and equinox of date, the Greenwich Apparent Sidereal Time and the polar motion matrix. One use of the present routine is when generating a series of celestial-to-terrestrial matrices where only the Sidereal Time changes, avoiding the considerable overhead of recomputing the precession-nutation more often than necessary to achieve given accuracy objectives.

2. The matrix RC2T transforms from celestial to terrestrial coordinates:

\[ v_{TRS} = R_{POM} R_3(ERA) R_{C2I} v_{CRS} \]
\[ \equiv R_{C2T} v_{CRS} \]

where $v_{TRS}$ is a vector in the International Terrestrial Reference System (see IERS Conventions 2003), $R_{POM}$ is the polar motion matrix, $ERA$ is the Earth Rotation Angle, $R_{C2I}$ is the CIO based bias-precession-nutation matrix, $v_{CRS}$ is a vector in the Geocentric Celestial Reference System, and $R_{C2T}$ is the RC2T matrix returned by the present routine.

REFERENCE:

CALL:  
CALL iau_C2TPE (TTA, TTB, UTA, UTB, DPSI, DEPS, XP, YP),

ACTION:  
Form the celestial to terrestrial matrix given the date, the UT1, the nutation and the polar motion. IAU 2000.

GIVEN:  
TTA, TTB \(d\)  TT as a two-part Julian Date (Note 1)  
UTA, UTB \(d\)  UT1 as a two-part Julian Date (Note 1)  
DPSI, DEPS \(d\)  nutation (Note 2)  
XP, YP \(d\)  coordinates of the pole (radians, Note 3)

RETURNED:  
RC2T \((3,3)\)  celestial-to-terrestrial matrix (Note 4)

NOTES:  
1. The TT and UT1 dates \(TTA+TTB\) and \(UTA+UTB\) are Julian Dates, apportioned in any convenient way between the arguments \(UTA\) and \(UTB\). For example, 
\[JD(UT1) = 2450123.7\] could be expressed in any of these ways, among others:

<table>
<thead>
<tr>
<th>UTA</th>
<th>UTB</th>
</tr>
</thead>
<tbody>
<tr>
<td>2450123.7D0</td>
<td>0D0</td>
</tr>
<tr>
<td>2451545.0D0</td>
<td>−1421.3D0</td>
</tr>
<tr>
<td>2400000.5D0</td>
<td>50123.2D0</td>
</tr>
<tr>
<td>2450123.5D0</td>
<td>0.2D0</td>
</tr>
</tbody>
</table>

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 and MJD methods are good compromises between resolution and convenience. In the case of \(UTA, UTB\), the date & time method is best matched to the Earth rotation angle algorithm used: maximum accuracy (or, at least, minimum noise) is delivered when the \(UTA\) argument is for \(0^h\) UT1 on the day in question and the \(UTB\) argument lies in the range \(0\) to \(1\), or \textit{vice versa}.

2. The caller is responsible for providing the nutation components; they are in longitude and obliquity, in radians and are with respect to the equinox and ecliptic of date. For high-accuracy applications, free core nutation should be included as well as any other relevant corrections to the position of the CIP.

3. XP and YP are the coordinates (in radians) of the Celestial Intermediate Pole with respect to the International Terrestrial Reference System (see IERS Conventions 2003), measured along the meridians 0° and 90° west respectively.
4. The matrix $\mathbf{RC2T}$ transforms from celestial to terrestrial coordinates:

$$
\mathbf{v}_{TRS} = \mathbf{R}_{POM} \mathbf{R}_3(GST) \mathbf{R}_{BNP} \mathbf{v}_{CRS} = \mathbf{R}_{C2T} \mathbf{v}_{CRS}
$$

where $\mathbf{v}_{TRS}$ is a vector in the International Terrestrial Reference System (see IERS Conventions 2003), $\mathbf{R}_{POM}$ is the polar motion matrix, $GST$ is the Greenwich (apparent) Sidereal Time, $\mathbf{R}_{BNP}$ is the equinox based bias-precession-nutation matrix, $\mathbf{v}_{CRS}$ is a vector in the Geocentric Celestial Reference System, and $\mathbf{R}_{C2T}$ is the $\mathbf{RC2T}$ matrix returned by the present routine.

5. Although its name does not include “00”, this routine is in fact specific to the IAU 2000 models.

**REFERENCE**:

CALL:
CALL iau_C2TXY ( TTA, TTB, UTA, UTB, X, Y, XP, YP, RC2T )

ACTION:
Form the celestial to terrestrial matrix given the date, the UT1, the CIP X,Y coordinates and the polar motion. IAU 2000.

GIVEN:

<table>
<thead>
<tr>
<th>TTA,TTB</th>
<th>d</th>
<th>TT as a two-part Julian Date (Note 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UTA,UTB</td>
<td>d</td>
<td>UT1 as a two-part Julian Date (Note 1)</td>
</tr>
<tr>
<td>X,Y</td>
<td>d</td>
<td>Celestial Intermediate Pole (Note 2)</td>
</tr>
<tr>
<td>XP,YP</td>
<td>d</td>
<td>coordinates of the pole (radians, Note 3)</td>
</tr>
</tbody>
</table>

RETURNED:

| RC2T     | d(3,3) | celestial-to-terrestrial matrix (Note 4) |

NOTES:

1. The TT and UT1 dates TTA+TTB and UTA+UTB are Julian Dates, apportioned in any convenient way between the arguments UTA and UTB. For example,
   JD(UT1) = 2450123.7 could be expressed in any of these ways, among others:
   
<table>
<thead>
<tr>
<th>UTA</th>
<th>UTB</th>
</tr>
</thead>
<tbody>
<tr>
<td>2450123.7D0</td>
<td>0D0</td>
</tr>
<tr>
<td>2451545.0D0</td>
<td>-1421.3D0</td>
</tr>
<tr>
<td>2400000.5D0</td>
<td>50123.2D0</td>
</tr>
<tr>
<td>2450123.5D0</td>
<td>0.2D0</td>
</tr>
</tbody>
</table>

   The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 and MJD methods are good compromises between resolution and convenience. In the case of UTA,UTB, the date & time method is best matched to the Earth rotation angle algorithm used: maximum accuracy (or, at least, minimum noise) is delivered when the UTA argument is for 0h UT1 on the day in question and the UTB argument lies in the range 0 to 1, or vice versa.

2. The Celestial Intermediate Pole coordinates X,Y are the x,y components of the unit vector in the Geocentric Celestial Reference System.

3. XP and YP are the coordinates (in radians) of the Celestial Intermediate Pole with respect to the International Terrestrial Reference System (see IERS Conventions 2003), measured along the meridians 0° and 90° west respectively.
4. The matrix $\text{RC2T}$ transforms from celestial to terrestrial coordinates:

$$
\mathbf{v}_{\text{TRS}} = \mathbf{R}_{\text{POM}} \mathbf{R}_3(\text{GST}) \mathbf{R}_{\text{BPN}} \mathbf{v}_{\text{CRS}}
$$

$$
= \mathbf{R}_{\text{C2T}} \mathbf{v}_{\text{CRS}}
$$

where $\mathbf{v}_{\text{TRS}}$ is a vector in the International Terrestrial Reference System (see IERS Conventions 2003), $\mathbf{R}_{\text{POM}}$ is the polar motion matrix, $\text{GST}$ is the Greenwich (apparent) Sidereal Time, $\mathbf{R}_{\text{BPN}}$ is the equinox based bias-precession-nutation matrix, $\mathbf{v}_{\text{CRS}}$ is a vector in the Geocentric Celestial Reference System, and $\mathbf{R}_{\text{C2T}}$ is the $\text{RC2T}$ matrix returned by the present routine.

5. Although its name does not include “00”, this routine is in fact specific to the IAU 2000 models.

REFERENCE :

CALL:

CALL iau_CAL2JD ( IY, IM, ID, DJM0, DJM, J )

ACTION:

Gregorian Calendar to Julian Date.

GIVEN:

\( IY, IM, ID \)  
year, month, day in Gregorian calendar (Note 1)

RETURNED:

\( DJM0 \)  
MJD zero-point: always 2400000.5D0

\( DJM \)  
Modified Julian Date for 0h

\( J \)  
status: 0 = OK

-1 = bad year (Note 3; JD not computed)
-2 = bad month (JD not computed)
-3 = bad day (JD computed)

NOTES:

1. The algorithm used is valid from \(-4800\) March 1, but this implementation rejects dates before \(-4799\) January 1.

2. The Julian Date is returned in two pieces, in the usual SOFA manner, which is designed to preserve time resolution. The Julian Date is available as a single number by adding \( DJM0 \) and \( DJM \).

3. In early eras the conversion is from the “Proleptic Gregorian Calendar”; no account is taken of the date(s) of adoption of the Gregorian Calendar, nor is any AD/BC numbering convention observed.

REFERENCE:

CALL :  
    CALL iau_CR ( R, C )

ACTION :  
    Copy an r-matrix.

GIVEN :  
    \( R \) d(3,3) r-matrix to be copied

RETURNED :  
    \( C \) d(3,3) copy
**iau_DAT**  
*calculate TAI–UTC*

**CALL:**

CALL iau_DAT ( IY, IM, ID, FD, DELTAT, J )

**ACTION:**

For a given UTC date, calculate $\Delta AT = \text{TAI} - \text{UTC}$.

**GIVEN:**

$IY$  
i  
UTC: year (Notes 1 and 2)

$IM$  
i  
month (Note 2)

$ID$  
i  
day (Notes 2 and 3)

$FD$  
d  
fraction of day (Note 4)

**RETURNED:**

$DELTAT$  
d  
TAI minus UTC, seconds

$J$  
i  
status:  
$+1$ = dubious year (Note 1)  
$0$ = OK  
$-1$ = bad year  
$-2$ = bad month  
$-3$ = bad day (Note 3)  
$-4$ = bad fraction (Note 4)  
$-5$ = internal error (Note 5)

**NOTES:**

1. UTC began at 1960 January 1.0 (JD 2436934.5) and it is improper to call the routine with an earlier date. If this is attempted, zero is returned together with a warning status.

Because leap seconds cannot, in principle, be predicted in advance, a reliable check for dates beyond the valid range is impossible. To guard against gross errors, a year five or more after the release year of the present routine (see parameter $IYV$) is considered dubious. In this case a warning status is returned but the result is computed in the normal way.

For both too-early and too-late years, the warning status is $J = +1$. This is distinct from the error status $J = -1$, which signifies a year so early that JD could not be computed.

2. If the specified date is for a day which ends with a leap second, the TAI–UTC value returned is for the period leading up to the leap second. If the date is for a day which begins as a leap second ends, the TAI–UTC returned is for the period following the leap second.
3. The day number must be in the normal calendar range, for example 1 through 30 for April. The “almanac” convention of allowing such dates as January 0 and December 32 is not supported in this routine, in order to avoid confusion near leap seconds.

4. The fraction of day is used only for dates before the introduction of leap seconds, the first of which occurred at the end of 1971. It is tested for validity (0 to 1 is the valid range) even if not used; if invalid, zero is used and status $J = -4$ is returned. For many applications, setting $FD$ to zero is acceptable; the resulting error is always less than 3 ms (and occurs only pre-1972).

5. The status value returned in the case where there are multiple errors refers to the first error detected. For example, if the month and day are 13 and 32 respectively, $J = -2$ (bad month) will be returned. The “internal error” status refers to a case that is impossible but causes some compilers to issue a warning.

6. In cases where a valid result is not available, zero is returned.

REFERENCES:

1. For dates from 1961 January 1 onwards, the expressions from the file ftp://maia.usno.navy.mil/ser7/TAI-UTC.dat are used.

CALL:
\[ D = \text{iau\_EE00}(\text{DATE1}, \text{DATE2}, \text{EPSA}, \text{DPSI}) \]

**ACTION:**
The equation of the equinoxes, compatible with IAU 2000 resolutions, given the nutation in longitude and the mean obliquity.

**GIVEN:**
- \( \text{DATE1} \): TT as a two-part...
- \( \text{DATE2} \): ...Julian Date (Note 1)
- \( \text{EPSA} \): mean obliquity (Note 2)
- \( \text{DPSI} \): mutation in longitude (Note 3)

**RETURNED** (function value):
\[ d \quad \text{equation of the equinoxes (Note 4)} \]

**NOTES:**
1. The TT date \( \text{DATE1}+\text{DATE2} \) is a Julian Date, apportioned in any convenient way between the arguments \( \text{DATE1} \) and \( \text{DATE2} \). For example, \( \text{JD}(\text{TT}) = 2450123.7 \) could be expressed in any of these ways, among others:
   - \( \text{DATE1} \): 2450123.7D0, \( \text{DATE2} \): 0D0 (JD method)
   - \( \text{DATE1} \): 2451545.0D0, \( \text{DATE2} \): -1421.3D0 (J2000 method)
   - \( \text{DATE1} \): 2400000.5D0, \( \text{DATE2} \): 50123.2D0 (MJD method)
   - \( \text{DATE1} \): 2450123.5D0, \( \text{DATE2} \): 0.2D0 (date & time method)

   The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The obliquity, in radians, is mean of date.

3. The result, which is in radians, operates in the following sense:
   
   Greenwich apparent \( ST = \text{GMST} + \text{equation of the equinoxes} \)

4. The result is compatible with the IAU 2000 resolutions. For further details, see IERS Conventions 2003 and Capitaine et al. (2002).

**REFERENCES:**
iau_EE00A  equation of the equinoxes, IAU 2000A

CALL:
\[ D = \text{iau\_EE00A}(\text{DATE1}, \text{DATE2}) \]

ACTION:
Equation of the equinoxes, compatible with IAU 2000 resolutions.

GIVEN:
\begin{align*}
\text{DATE1} & \quad d & \text{TT as a two-part} \\
\text{DATE2} & \quad d & \text{...Julian Date (Note 1)}
\end{align*}

RETURNED (function value):
\[ d \quad \text{equation of the equinoxes (Note 2)} \]

NOTES:
1. The TT date \text{DATE1+DATE2} is a Julian Date, apportioned in any convenient way between the arguments \text{DATE1} and \text{DATE2}. For example, \text{JD(TT)} = 2450123.7 could be expressed in any of these ways, among others:
\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 & \text{(JD method)} \\
2451545.0D0 & \quad -1421.3D0 & \text{(J2000 method)} \\
2400000.5D0 & \quad 50123.2D0 & \text{(MJD method)} \\
2450123.5D0 & \quad 0.2D0 & \text{(date & time method)}
\end{align*}
The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The result, which is in radians, operates in the following sense:
   \[ \text{Greenwich apparent ST} = \text{GMST} + \text{equation of the equinoxes} \]

3. The result is compatible with the IAU 2000 resolutions. For further details, see IERS Conventions 2003 and Capitaine et al. (2002).

4. A faster, but slightly less accurate, result (about 1 mas) can be obtained by using instead the \text{iau\_EE00B} routine.

REFERENCES:
CALL:
D = iau_EE00B (DATE1, DATE2)

ACTION:
Equation of the equinoxes, compatible with IAU 2000 resolutions but using the truncated nutation model IAU 2000B.

GIVEN:

DATE1  d    TT as a two-part...
DATE2  d    ...Julian Date (Note 1)

RETURNED (function value):

d  equation of the equinoxes (Note 2)

NOTES:

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

\[
\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 \quad (\text{JD method}) \\
2451545.0D0 & \quad -1421.3D0 \quad (\text{J2000 method}) \\
2400000.5D0 & \quad 50123.2D0 \quad (\text{MJD method}) \\
2450123.5D0 & \quad 0.2D0 \quad (\text{date & time method})
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The result, which is in radians, operates in the following sense:

Greenwich apparent ST = GMST + equation of the equinoxes

3. The result is compatible with the IAU 2000 resolutions except that accuracy has been compromised (1 mas) for the sake of speed. For further details, see McCarthy & Luzum (2003), IERS Conventions 2003 and Capitaine et al. (2003).

REFERENCES:

**CALL:**

\[ D = \text{iau\_EE06A}(\text{DATE1}, \text{DATE2}) \]

**ACTION:**


**GIVEN:**

\[
\begin{align*}
\text{DATE1} & \quad \text{d} \quad \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} \quad \ldots\text{Julian Date (Note 1)}
\end{align*}
\]

**RETURNED (function value):**

\[ d \quad \text{equation of the equinoxes (Note 2)} \]

**NOTES:**

1. The TT date \( \text{DATE1+DATE2} \) is a Julian Date, apportioned in any convenient way between the arguments \( \text{DATE1} \) and \( \text{DATE2} \). For example, \( \text{JD}(\text{TT}) = 2450123.7 \) could be expressed in any of these ways, among others:

\[
\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 \quad \text{(JD method)} \\
2451545.0D0 & \quad -1421.3D0 \quad \text{(J2000 method)} \\
2400000.5D0 & \quad 50123.2D0 \quad \text{(MJD method)} \\
2450123.5D0 & \quad 0.2D0 \quad \text{(date & time method)}
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The result, which is in radians, operates in the following sense:

\[
\text{Greenwich apparent ST} = \text{GMST} + \text{equation of the equinoxes}
\]

**REFERENCE:**

CALL:

\[ D = \text{iau}_\text{EECT00} \left( \text{DATE1}, \text{DATE2} \right) \]

ACTION:

Equation of the equinoxes complementary terms, consistent with IAU 2000 resolutions.

GIVEN:

\[ \text{DATE1} \quad \text{d} \quad \text{TT as a two-part...} \]
\[ \text{DATE2} \quad \text{d} \quad \ldots \text{Julian Date (Note 1)} \]

RETURNED (function value):

\[ \text{d} \quad \text{complementary terms (Note 2)} \]

NOTES:

1. The TT date \( \text{DATE1+DATE2} \) is a Julian Date, apportioned in any convenient way between the arguments \( \text{DATE1} \) and \( \text{DATE2} \). For example, \( \text{JD(TT)} = 2450123.7 \) could be expressed in any of these ways, among others:

\[
\begin{array}{c|c|c}
\text{DATE1} & \text{DATE2} & \text{method} \\
2450123.7D0 & 0D0 & \text{JD method} \\
2451545.0D0 & -1421.3D0 & \text{J2000 method} \\
2400000.5D0 & 50123.2D0 & \text{MJD method} \\
2450123.5D0 & 0.2D0 & \text{date & time method} \\
\end{array}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The “complementary terms” are part of the equation of the equinoxes (EE), classically the difference between apparent and mean Sidereal Time:

\[ \text{GAST} = \text{GMST} + \text{EE} \]

with:

\[ \text{EE} = \Delta \psi \cos \epsilon \]

where \( \Delta \psi \) is the nutation in longitude and \( \epsilon \) is the obliquity of date. However, if the rotation of the Earth were constant in an inertial frame the classical formulation would lead to apparent irregularities in the UT1 time scale traceable to side-effects of precession-nutation. In order to eliminate these effects from UT1, “complementary terms” were introduced in 1994 (IAU, 1994) and took effect from 1997 (Capitaine & Gontier, 1993):
GAST = GMST + CT + EE

By convention, the complementary terms are included as part of the equation of the equinoxes rather than as part of the mean Sidereal Time. This slightly compromises the “geometrical” interpretation of mean sidereal time but is otherwise inconsequential.

The present routine computes CT in the above expression, compatible with IAU 2000 resolutions (Capitaine et al., 2002, and IERS Conventions 2003).

REFERENCES:

CALL:
D = iau_EO06A (DATE1, DATE2)

ACTION:

GIVEN:

| DATE1 | d | TT as a two-part... |
| DATE2 | d | ...Julian Date (Note 1) |

RETURNED (function value):

d the equation of the origins in radians

NOTES:

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

<table>
<thead>
<tr>
<th>DATE1</th>
<th>DATE2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2450123.7D0</td>
<td>0D0</td>
</tr>
<tr>
<td>2451545.0D0</td>
<td>-1421.3D0</td>
</tr>
<tr>
<td>2400000.5D0</td>
<td>50123.2D0</td>
</tr>
<tr>
<td>2450123.5D0</td>
<td>0.2D0</td>
</tr>
</tbody>
</table>

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The equation of the origins is the distance between the true equinox and the celestial intermediate origin and, equivalently, the difference between Earth rotation angle and Greenwich apparent sidereal time (ERA−GST). It comprises the precession (since J2000.0) in right ascension plus the equation of the equinoxes (including the small correction terms).

REFERENCES:

**iau_EORS**  
*equation of the origins from NPB matrix and s*

**CALL**:

D = iau_EORS (RNPB, S)

**ACTION**:

Equation of the origins, given the classical NPB matrix and the quantity s.

**GIVEN**:

- **RNPB**
  - d(3,3) classical nutation × precession × bias matrix
- **S**
  - d the quantity s (the CIO locator) in radians

**RETURNED** (function value):

- d the equation of the origins in radians

**NOTES**:

1. The equation of the origins is the distance between the true equinox and the celestial intermediate origin and, equivalently, the difference between Earth rotation angle and Greenwich apparent sidereal time (ERA−GST). It comprises the precession (since J2000.0) in right ascension plus the equation of the equinoxes (including the small correction terms).

2. The algorithm is from Wallace & Capitaine (2006).

**REFERENCES**:

CALL:

\[ D = \text{iau\_EQEQ94}(\text{DATE1, DATE2}) \]

ACTION:

Equation of the equinoxes, IAU 1994 model.

GIVEN:

\[
\begin{align*}
\text{DATE1} & \quad \text{d} \quad \text{TDB as a two-part...} \\
\text{DATE2} & \quad \text{d} \quad \text{...Julian Date (Note 1)}
\end{align*}
\]

RETURNED (function value):

\[
\text{d} \quad \text{equation of the equinoxes (Note 2)}
\]

NOTES:

1. The TDB date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TDB) = 2450123.7 could be expressed in any of these ways, among others:

\[
\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 \quad \text{(JD method)} \\
2451545.0D0 & \quad -1421.3D0 \quad \text{(J2000 method)} \\
2400000.5D0 & \quad 50123.2D0 \quad \text{(MJD method)} \\
2450123.5D0 & \quad 0.2D0 \quad \text{(date & time method)}
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The result, which is in radians, operates in the following sense:

\[
\text{Greenwich apparent ST} = \text{GMST} + \text{equation of the equinoxes}
\]

REFERENCES:

**CALL**:  
\[ D = \text{iau\_ERA00}(\text{DJ1}, \text{DJ2}) \]

**ACTION**:  
Earth rotation angle (IAU 2000 model).

**GIVEN**:

\[
\begin{align*}
\text{DJ1} & \quad \text{d} \quad \text{UT1 as a two-part...} \\
\text{DJ2} & \quad \text{d} \quad \ldots\text{Julian Date (Note 1)}
\end{align*}
\]

**RETURNED** (function value):

\[
\begin{align*}
\text{d} & \quad \text{ERA in radians (Note 2)}
\end{align*}
\]

**NOTES**:

1. The UT1 date DJ1+DJ2 is a Julian Date, apportioned in any convenient way between the arguments DJ1 and DJ2. For example, JD(UT1) = 2450123.7 could be expressed in any of these ways, among others:

\[
\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 \quad (\text{JD method}) \\
2451545.0D0 & \quad -1421.3D0 \quad (\text{J2000 method}) \\
2400000.5D0 & \quad 50123.2D0 \quad (\text{MJD method}) \\
2450123.5D0 & \quad 0.2D0 \quad (\text{date & time method})
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 and MJD methods are good compromises between resolution and convenience. The date & time method is best matched to the algorithm used: maximum accuracy (or, at least, minimum noise) is delivered when the DJ1 argument is for 0h UT1 on the day in question and the DJ2 argument lies in the range 0 to 1, or *vice versa*.

2. The algorithm is adapted from Expression 22 of Capitaine *et al.* 2000. The time argument has been expressed in days directly, and, to retain precision, integer contributions have been eliminated. The same formulation is given in IERS Conventions (2003), Chap. 5, Eq. 14.

**REFERENCES**:

\texttt{iau\_FAD03} \hspace{1cm} mean elongation of Moon from Sun, \textit{IERS 2003} \hspace{1cm} \texttt{iau\_FAD03}

\textbf{CALL}:

\[ D = \text{iau\_FAD03}(T) \]

\textbf{ACTION}:


\textbf{GIVEN}:

\[ T \quad d \]

TDB, Julian centuries since J2000.0 (Note 1)

\textbf{RETURNED (function value)}:

\[ d \]

mean elongation of Moon from Sun, radians (Note 2)

\textbf{NOTES}:

1. Though \(T\) is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.

2. The expression used is as adopted in \textit{IERS Conventions (2003)} and is from Simon \textit{et al.} (1994).

\textbf{REFERENCES}:


CALL:
   D = iau_FAE03 ( T )

ACTION:

GIVEN:
   T  d  TDB, Julian centuries since J2000.0 (Note 1)

RETURNED (function value):
   d  mean longitude of Earth, radians (Note 2)

NOTES:
1. Though T is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.
2. The expression used is as adopted in IERS Conventions (2003) and comes from Souchay et al. (1999) after Simon et al. (1994).

REFERENCES:
CALL:
D = iau_FAF03 ( T )

ACTION:
Fundamental argument, IERS Conventions (2003): mean longitude of the Moon minus mean longitude of the ascending node.

GIVEN:
T d
TDB, Julian centuries since J2000.0 (Note 1)

RETURNED (function value):
d mean long. of Moon minus asc. node, radians (Note 2)

NOTES:
1. Though T is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.
2. The expression used is as adopted in IERS Conventions (2003) and is from Simon et al. (1994).

REFERENCES:
**iau**\_**FAJU03** \hspace{1cm} \textit{mean longitude of Jupiter, IERS 2003} \hspace{1cm} **iau**\_**FAJU03**

**CALL** :

\[ D = \text{iau}\_\text{FAJU03}(T) \]

**ACTION** :


**GIVEN** :

\[ T \]

TDB, Julian centuries since J2000.0 (Note 1)

**RETURNED (function value)** :

\[ d \]

mean longitude of Jupiter, radians (Note 2)

**NOTES** :

1. Though \( T \) is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.


**REFERENCES** :

CALL:
\[ D = \text{iau\_FAL03} \left( T \right) \]

ACTION:

GIVEN:
\( T \)  \( \text{TDB, Julian centuries since J2000.0 (Note 1)} \)

RETURNED (function value):
\( d \)  \( \text{mean anomaly of the Moon, radians (Note 2)} \)

NOTES:
1. Though \( T \) is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.

2. The expression used is as adopted in IERS Conventions (2003) and is from Simon et al. (1994).

REFERENCES:
CALL:
\[ D = \text{iau\_FALP03}(T) \]

ACTION:

GIVEN:
\[ T \]  
TDB, Julian centuries since J2000.0 (Note 1)

RETURNED (function value):
\[ d \]  
mean anomaly of the Sun, radians (Note 2)

NOTES:
1. Though \( T \) is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.

2. The expression used is as adopted in IERS Conventions (2003) and is from Simon \textit{et al}. (1994).

REFERENCES:
CALL:
\[ D = \text{iau\_FAMA03}(T) \]

ACTION:

GIVEN:
\[ T \]  
TDB, Julian centuries since J2000.0 (Note 1)

RETURNED (function value):
\[ d \]  
mean longitude of Mars, radians (Note 2)

NOTES:
1. Though \( T \) is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.

2. The expression used is as adopted in IERS Conventions (2003) and comes from Souchay et al. (1999) after Simon et al. (1994).

REFERENCES:
CALL:

\[ D = \text{iau\_FAME03} \left( T \right) \]

ACTION:

GIVEN:
\[ T \quad \text{TDB, Julian centuries since J2000.0} \quad (\text{Note 1}) \]

RETURNED (function value):
\[ d \quad \text{mean longitude of Mercury, radians} \quad (\text{Note 2}) \]

NOTES:
1. Though \( T \) is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.

2. The expression used is as adopted in IERS Conventions (2003) and comes from Souchay et al. (1999) after Simon et al. (1994).

REFERENCES:
CALL:
   \[ D = \text{iau}_F\text{ANE03} ( T ) \]

ACTION:

GIVEN:
   \[ T \]
   \[ d \]
   TDB, Julian centuries since J2000.0 (Note 1)

RETURNED (function value):
   \[ d \]
   mean longitude of Neptune, radians (Note 2)

NOTES:
   1. Though \( T \) is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.
   2. The expression used is as adopted in IERS Conventions (2003) and comes from Souchay et al. (1999) after Simon et al. (1994).

REFERENCES:
CALL:

\[ D = \text{iau\_FAOM03} \left( T \right) \]

ACTION:

Fundamental argument, IERS Conventions (2003): mean longitude of the Moon’s ascending node.

GIVEN:

\[ T \quad d \]

TDB, Julian centuries since J2000.0 (Note 1)

RETURNED (function value):

\[ d \]

mean long. of Moon’s asc. node, radians (Note 2)

NOTES:

1. Though \( T \) is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.

2. The expression used is as adopted in IERS Conventions (2003) and is from Simon et al. (1994).

REFERENCES:


iau_FAPA03  \hspace{1cm} \textit{general precession in longitude, IERS 2003} \hspace{1cm} \text{iau_FAPA03}

**CALL:**
\[ D = \text{iau_FAPA03} \left( T \right) \]

**ACTION:**

**GIVEN:**
\[ T \]  
\[ d \]  
TDB, Julian centuries since J2000.0 (Note 1)

**RETURNED** (function value):
\[ d \]  
general precession in longitude, radians (Note 2)

**NOTES:**
1. Though \( T \) is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.
2. The expression used is as adopted in IERS Conventions (2003). It is taken from Kinoshita & Souchay (1990) and comes originally from Lieske \textit{et al.} (1977).

**REFERENCES:**
**CALL**:  
\[ D = \text{iau}_FASA03 \left( T \right) \]

**ACTION**:  

**GIVEN**:  
\( T \)  
TDB, Julian centuries since J2000.0 (Note 1)

**RETURNED** (function value) :  
\( d \)  
mean longitude of Saturn, radians (Note 2)

**NOTES**:  
1. Though \( T \) is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.
2. The expression used is as adopted in IERS Conventions (2003) and comes from Souchay et al. (1999) after Simon et al. (1994).

**REFERENCES**:  
\textbf{CALL :} \\
\texttt{D = iau\_FAUR03 ( T )}

\textbf{ACTION :} \\

\textbf{GIVEN :} \\
\texttt{T} \quad \texttt{d} \quad \text{TDB, Julian centuries since J2000.0 (Note 1)}

\textbf{RETURNED (function value) :} \\
\texttt{d} \quad \text{mean longitude of Uranus, radians (Note 2)}

\textbf{NOTES :} \\
1. Though \texttt{T} is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.


\textbf{REFERENCES :} \\


**CALL:**

\[ D = \text{iau\_FAVE03} (T) \]

**ACTION:**


**GIVEN:**

\[ T \quad \text{d} \]

TDB, Julian centuries since J2000.0 (Note 1)

**RETURNED (function value):**

\[ d \]

mean longitude of Venus, radians (Note 2)

**NOTES:**

1. Though \( T \) is strictly TDB, it is usually more convenient to use TT, which makes no significant difference.

2. The expression used is as adopted in IERS Conventions (2003) and comes from Souchay et al. (1999) after Simon et al. (1994).

**REFERENCES:**


CALL:

CALL iau_FW2M ( GAMB, PHIB, PSI, EPS, R )

ACTION:

Form rotation matrix given the Fukushima-Williams angles.

GIVEN:

\begin{align*}
\text{GAMB} & \quad \text{d} \quad \text{F-W angle } \gamma (\text{radians}) \\
\text{PHIB} & \quad \text{d} \quad \text{F-W angle } \phi (\text{radians}) \\
\text{PSI} & \quad \text{d} \quad \text{F-W angle } \psi (\text{radians}) \\
\text{EPS} & \quad \text{d} \quad \text{F-W angle } \epsilon (\text{radians})
\end{align*}

RETURNED:

\begin{align*}
R & \quad \text{d}(3,3) \quad \text{rotation matrix}
\end{align*}

NOTES:

1. Naming the following points:

\begin{align*}
e & = \text{J}2000.0 \text{ ecliptic pole,} \\
p & = \text{GCRS pole,} \\
E & = \text{ecliptic pole of date and} \\
P & = \text{CIP,}
\end{align*}

the four Fukushima-Williams angles are as follows:

\begin{align*}
\gamma & = e p E \\
\phi & = p E \\
\psi & = p E P \\
\epsilon & = E P
\end{align*}

2. The matrix representing the combined effects of frame bias, precession and nutation is:

\[
\mathbf{N P B} = R_1(-\epsilon) R_3(-\psi) R_1(\dot{\phi}) R_3(\dot{\gamma})
\]

3. The present routine can construct three different matrices, depending on which angles are supplied as the arguments GAMB, PHIB, PSI and EPS:

- To obtain the \textbf{N P B} matrix, first generate the four precession angles known conventionally as \( \gamma, \phi, \psi \) and \( \epsilon_A \), then generate the nutation components \( \Delta \psi \) and \( \Delta \epsilon \) and add them to \( \dot{\psi} \) and \( \dot{\epsilon} \), and finally call the present routine using those four angles as arguments.
- To obtain the \textbf{P B} matrix, generate the four precession angles and call the present routine.
- To obtain the \textbf{B} matrix, generate the four precession angles for date J2000.0 and call the present routine.

The nutation-only and precession-only matrices can if necessary be obtained by combining these three appropriately.
REFERENCES:

CALL:
CALL iau_FW2XY ( GAMB, PHIB, PSI, EPS, X, Y )

ACTION:
CIP X, Y given Fukushima-Williams bias-precession-nutation angles.

GIVEN:
\begin{align*}
GAMB & \quad \text{d} \quad \text{F-W angle } \bar{\gamma} \text{ (radians)} \\
PHIB & \quad \text{d} \quad \text{F-W angle } \bar{\phi} \text{ (radians)} \\
PSI & \quad \text{d} \quad \text{F-W angle } \psi \text{ (radians)} \\
EPS & \quad \text{d} \quad \text{F-W angle } \epsilon \text{ (radians)}
\end{align*}

RETURNED:
\begin{align*}
X, Y & \quad \text{d} \quad \text{CIP unit vector } X, Y
\end{align*}

NOTES:
1. Naming the following points:
   \begin{align*}
   e &= \text{J2000.0 ecliptic pole}, \\
p &= \text{GCRS pole}, \\
E &= \text{ecliptic pole of date and} \\
P &= \text{CIP},
\end{align*}
   the four Fukushima-Williams angles are as follows:
   \begin{align*}
   \bar{\gamma} &= epE \\
\bar{\phi} &= pE \\
\psi &= pEP \\
\epsilon &= EP
\end{align*}

2. The matrix representing the combined effects of frame bias, precession and nutation is:
\[ NPB = R_1(-\epsilon) R_3(-\psi) R_1(\bar{\phi}) R_3(\bar{\gamma}) \]

   The returned values X, Y are elements (3,1) and (3,2) of the matrix. Near J2000.0, they are essentially angles in radians.

REFERENCE:
**iau_GMST00** *Greenwich Mean Sidereal Time, IAU 2000*

**CALL:**

\[ D = \text{iau}_\text{GMST00} \left( UTA, UTB, TTA, TTB \right) \]

**ACTION:**

Greenwich Mean Sidereal Time (model consistent with IAU 2000 resolutions).

**GIVEN:**

\begin{align*}
UTA, UTB & \quad \text{d} \quad \text{UT1 as a two-part Julian Date (Notes 1,2)} \\
TTA, TTB & \quad \text{d} \quad \text{TT as a two-part Julian Date (Notes 1,2)}
\end{align*}

**RETURNED** (function value):

\[ d \quad \text{Greenwich mean sidereal time (radians)} \]

**NOTES:**

1. The UT1 and TT dates \( UTA + UTB \) and \( TTA + TTB \) respectively, are both Julian Dates, apportioned in any convenient way between the argument pairs. For example, \( JD(UT1) = 2450123.7 \) could be expressed in any of these ways, among others:

\[
\begin{array}{cc}
\text{UTA} & \text{UTB} \\
2450123.7D0 & 0D0 & \text{(JD method)} \\
2451545.0D0 & -1421.3D0 & \text{(J2000 method)} \\
2400000.5D0 & 50123.2D0 & \text{(MJD method)} \\
2450123.5D0 & 0.2D0 & \text{(date & time method)}
\end{array}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable (in the case of UT; the TT is not at all critical in this respect). The J2000 and MJD methods are good compromises between resolution and convenience. For UT, the date & time method is best matched to the algorithm that is used by the Earth Rotation Angle routine, called internally: maximum accuracy (or, at least, minimum noise) is delivered when the UTA argument is for 0° UT1 on the day in question and the UTB argument lies in the range 0 to 1, or vice versa.

2. Both UT1 and TT are required, UT1 to predict the Earth rotation and TT to predict the effects of precession. If UT1 is used for both purposes, errors of order 100µas result.

3. This GMST is compatible with the IAU 2000 resolutions and must be used only in conjunction with other IAU 2000 compatible components such as precession-nutation and equation of the equinoxes.

4. The result is returned in the range 0 to 2\( \pi \).


**REFERENCES:**


iau_GMST06  \textit{Greenwich mean sidereal time, IAU 2006}  iau_GMST06

CALL:
\[
D = \text{iau}_\text{GMST06} \left( \text{UTA}, \text{UTB}, \text{TTA}, \text{TTB} \right)
\]

ACTION:
Greenwich mean sidereal time (consistent with IAU 2006 precession).

GIVEN:
\begin{align*}
\text{UTA}, \text{UTB} & \quad \text{d} & \text{UT1 as a two-part Julian Date (Notes 1,2)} \\
\text{TTA}, \text{TTB} & \quad \text{d} & \text{TT as a two-part Julian Date (Notes 1,2)}
\end{align*}

RETURNED (function value):
\[
d \quad \text{Greenwich mean sidereal time (radians)}
\]

NOTES:
1. The UT1 and TT dates UT+UTB and TTA+TTB respectively, are both Julian Dates, apportioned in any convenient way between the argument pairs. For example, JD(UT1) = 2450123.7 could be expressed in any of these ways, among others:
\[
\begin{array}{ccc}
\text{UTA} & \text{UTB} & \text{UT1 as a two-part Julian Date (Notes 1,2)} \\
2450123.7D0 & 0D0 & (\text{JD method}) \\
2451545.0D0 & -1421.3D0 & (\text{J2000 method}) \\
2400000.5D0 & 50123.2D0 & (\text{MJD method}) \\
2450123.5D0 & 0.2D0 & (\text{date & time method})
\end{array}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable (in the case of UT; the TT is not at all critical in this respect). The J2000 and MJD methods are good compromises between resolution and convenience. For UT, the date & time method is best matched to the algorithm that is used by the Earth Rotation Angle routine, called internally: maximum accuracy (or, at least, minimum noise) is delivered when the \text{UTA} argument is for 0\text{h} UT1 on the day in question and the \text{UTB} argument lies in the range 0 to 1, or \textit{vice versa}.

2. Both UT1 and TT are required, UT1 to predict the Earth rotation and TT to predict the effects of precession. If UT1 is used for both purposes, errors of order 100\mu as result.

3. This GMST is compatible with the IAU 2006 precession and must not be used with other precession models.

4. The result is returned in the range 0 to 2\pi.

REFERENCE:
iau_GMST82  Greenwich Mean Sidereal Time, IAU 1982  iau_GMST82

CALL:
\[ D = iau_{\text{GMST82}}\left( DJ1, DJ2 \right) \]

ACTION:
Universal Time to Greenwich Mean Sidereal Time (IAU 1982 model).

GIVEN:
\[ DJ1, DJ2 \quad d \]
UT1 as a two-part Julian Date (see notes)

RETURNED (function value):
\[ d \]
Greenwich mean sidereal time (radians)

NOTES:
1. The UT1 DJ1+DJ2 is a Julian Date, apportioned in any convenient way between the two arguments. For example,
   \[ \text{JD(UT1)} = 2450123.7 \]
could be expressed in any of these ways, among others:

   \[
   \begin{array}{ccc}
   \text{DJ1} & \text{DJ2} \\
   2450123.7D0 & 0D0 & (\text{JD method}) \\
   2451545.0D0 & -1421.3D0 & (\text{J2000 method}) \\
   2400000.5D0 & 50123.2D0 & (\text{MJD method}) \\
   2450123.5D0 & 0.2D0 & (\text{date & time method})
   \end{array}
   \]

   The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable (in the case of UT; the TT is not at all critical in this respect). The J2000 and MJD methods are good compromises between resolution and convenience. For UT, the date & time method is best matched to the algorithm that is used by the Earth Rotation Angle routine, called internally: maximum accuracy (or, at least, minimum noise) is delivered when the DJ1 argument is for 0h UT1 on the day in question and the DJ2 argument lies in the range 0 to 1, or vice versa.

2. The algorithm is based on the IAU 1982 expression. This is always described as giving the GMST at 0h UT1. In fact, it gives the difference between the GMST and the UT1, the steady 4-minutes-per-day drawing-ahead of ST with respect to UT. When whole days are ignored, the expression happens to equal the GMST at 0h UT1 each day.

3. In this routine, the entire UT1 (the sum of the two arguments DJ1 and DJ2) is used directly as the argument for the standard formula, the constant term of which is adjusted by 12 hours to take account of the noon phasing of Julian Date. The UT1 is then added, but omitting whole days to conserve accuracy.

4. The result is returned in the range 0 to 2\pi.

REFERENCES:
iau_GST00A  Greenwich Apparent Sidereal Time, IAU 2000A  iau_GST00A

CALL :
\[ D = \text{iau\_GST00A} ( \text{UTA, UTB, TTA, TTB} ) \]

ACTION :
Greenwich Apparent Sidereal Time (consistent with IAU 2000 resolutions).

GIVEN :
\[ \text{UTA,UTB} \quad \text{d} \quad \text{UT1 as a two-part Julian Date (Notes 1,2)} \]
\[ \text{TTA,TTB} \quad \text{d} \quad \text{TT as a two-part Julian Date (Notes 1,2)} \]

RETURNED (function value) :
\[ \text{d} \quad \text{Greenwich apparent sidereal time (radians)} \]

NOTES :

1. The UT1 and TT dates \text{UTA+UTB} and \text{TTA+TTB} respectively, are both Julian Dates, apportioned in any convenient way between the argument pairs. For example, \text{JD(UT1) = 2450123.7} could be expressed in any of these ways, among others:

\[
\begin{align*}
\text{UTA} & \quad \text{UTB} \\
2450123.7\text{D0} & \quad 0\text{D0} \quad \text{(JD method)} \\
2451545.0\text{D0} & \quad -1421.3\text{D0} \quad \text{(J2000 method)} \\
2400000.5\text{D0} & \quad 50123.2\text{D0} \quad \text{(MJD method)} \\
2450123.5\text{D0} & \quad 0.2\text{D0} \quad \text{(date & time method)}
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable (in the case of UT; the TT is not at all critical in this respect). The J2000 and MJD methods are good compromises between resolution and convenience. For UT, the date & time method is best matched to the algorithm that is used by the Earth Rotation Angle routine, called internally: maximum accuracy (or, at least, minimum noise) is delivered when the \text{UTA} argument is for 0h UT1 on the day in question and the \text{UTB} argument lies in the range 0 to 1, or vice versa.

2. Both UT1 and TT are required, UT1 to predict the Earth rotation and TT to predict the effects of precession. If UT1 is used for both purposes, errors of order 100\mu as result.

3. This GAST is compatible with the IAU 2000 resolutions and must be used only in conjunction with other IAU 2000 compatible components such as precession-nutation.

4. The result is returned in the range 0 to 2\pi.

5. The algorithm is from Capitaine et al. (2003) and IERS Conventions 2003.

REFERENCES :

CALL:
\[ D = \text{iau\_GST00B} \left( \text{UTA}, \text{UTB} \right) \]

ACTION:

GIVEN:
\( \text{UTA,UTB} \)
\( d \) UT1 as a two-part Julian Date (Notes 1, 2)

RETURNED (function value):
\( d \) Greenwich apparent sidereal time (radians)

NOTES:

1. The UT1 date \( \text{UTA} + \text{UTB} \) is a Julian Date, apportioned in any convenient way between the two arguments. For example, \( \text{JD(UT1)} = 2450123.7 \) could be expressed in any of these ways, among others:

\[
\begin{align*}
\text{UTA} & = 2450123.7D0 \quad \text{UTB} = 0D0 & \text{(JD method)} \\
2451545.0D0 & - 1421.3D0 & \text{(J2000 method)} \\
2400000.5D0 & 50123.2D0 & \text{(MJD method)} \\
2450123.5D0 & 0.2D0 & \text{(date & time method)}
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable (in the case of UT; the TT is not at all critical in this respect). The J2000 and MJD methods are good compromises between resolution and convenience. For UT, the date & time method is best matched to the algorithm that is used by the Earth Rotation Angle routine, called internally: maximum accuracy (or, at least, minimum noise) is delivered when the \( \text{UTA} \) argument is for 0h UT1 on the day in question and the \( \text{UTB} \) argument lies in the range 0 to 1, or vice versa.

2. The result is compatible with the IAU 2000 resolutions, except that accuracy has been compromised for the sake of speed and convenience in two respects:

   • UT is used instead of TDB (or TT) to compute the precession component of GMST and the equation of the equinoxes. This results in errors of order 0.1 mas at present.
   • The IAU 2000B abridged nutation model (McCarthy & Luzum, 2003) is used, introducing errors of up to 1 mas.

3. This GAST is compatible with the IAU 2000 resolutions and must be used only in conjunction with other IAU 2000 compatible components such as precession-nutation.

4. The result is returned in the range 0 to \( 2\pi \).

5. The algorithm is from Capitaine et al. (2003) and IERS Conventions 2003.
REFERENCES:


CALL:
\[
D = \text{iau\_GST06} \left( \text{UTA}, \text{UTB}, \text{TTA}, \text{TTB}, \text{RNPB} \right)
\]

ACTION:
Greenwich apparent sidereal time, IAU 2006, given the NPB matrix.

GIVEN:
\begin{align*}
\text{UTA,UTB} & \quad \text{d} \quad \text{UT1 as a two-part Julian Date (Notes 1,2)} \\
\text{TTA,TTB} & \quad \text{d} \quad \text{TT as a two-part Julian Date (Notes 1,2)} \\
\text{RNPB} & \quad \text{d(3,3)} \quad \text{nutation} \times \text{precession} \times \text{bias matrix}
\end{align*}

RETURNED (function value):
\[
\text{d} \quad \text{Greenwich apparent sidereal time (radians)}
\]

NOTES:
1. The UT1 and TT dates $\text{UTA} + \text{UTB}$ and $\text{TTA} + \text{TTB}$ respectively, are both Julian Dates, apportioned in any convenient way between the argument pairs. For example, $\text{JD(UT1)} = 2450123.7$ could be expressed in any of these ways, among others:
\begin{align*}
\text{UTA} & \quad \text{d} \quad \text{UT1} \\
2450123.7D0 & \quad 0D0 \quad \text{(JD method)} \\
2451545.0D0 & \quad -1421.3D0 \quad \text{(J2000 method)} \\
2400000.5D0 & \quad 50123.2D0 \quad \text{(MJD method)} \\
2450123.5D0 & \quad 0.2D0 \quad \text{(date & time method)}
\end{align*}
The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable (in the case of UT; the TT is not at all critical in this respect). The J2000 and MJD methods are good compromises between resolution and convenience. For UT, the date & time method is best matched to the algorithm that is used by the Earth Rotation Angle routine, called internally: maximum accuracy (or, at least, minimum noise) is delivered when the $\text{UTA}$ argument is for $0^\text{h}$ UT1 on the day in question and the $\text{UTB}$ argument lies in the range 0 to 1, or vice versa.

2. Both UT1 and TT are required, UT1 to predict the Earth rotation and TT to predict the effects of precession-nutation. If UT1 is used for both purposes, errors of order 100$\mu$as result.

3. Although the routine uses the IAU 2006 series for $s + XY/2$, it is otherwise independent of the precession-nutation model and can in practice be used with any equinox-based NPB matrix.

4. The result is returned in the range 0 to $2\pi$.

REFERENCE:
**iau_GST06A** *Greenwich Appt. Sidereal Time, IAU 2006/2000A*

**CALL:**

\[ D = \text{iau\_GST06A}(\text{UTA, UTB, TTA, TTB}) \]

**ACTION:**

Greenwich apparent sidereal time (consistent with IAU 2000 and 2006 resolutions).

**GIVEN:**

- \( UTA, UTB \) \( d \) : UT1 as a two-part Julian Date (Notes 1,2)
- \( TTA, TTB \) \( d \) : TT as a two-part Julian Date (Notes 1,2)

**RETURNED** (function value):

\( d \) : Greenwich apparent sidereal time (radians)

**NOTES:**

1. The UT1 and TT dates \( UTA + UTB \) and \( TTA + TTB \) respectively, are both Julian Dates, apportioned in any convenient way between the argument pairs. For example, \( \text{JD}(UT1) = 2450123.7 \) could be expressed in any of these ways, among others:

<table>
<thead>
<tr>
<th>UTA</th>
<th>UTB</th>
</tr>
</thead>
<tbody>
<tr>
<td>2450123.7D0</td>
<td>0D0</td>
</tr>
<tr>
<td>2451545.0D0</td>
<td>−1421.3D0</td>
</tr>
<tr>
<td>2400000.5D0</td>
<td>50123.2D0</td>
</tr>
<tr>
<td>2450123.5D0</td>
<td>0.2D0</td>
</tr>
</tbody>
</table>

   The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable (in the case of UT; the TT is not at all critical in this respect). The J2000 and MJD methods are good compromises between resolution and convenience. For UT, the date & time method is best matched to the algorithm that is used by the Earth Rotation Angle routine, called internally: maximum accuracy (or, at least, minimum noise) is delivered when the \( \text{UTA} \) argument is for 0h UT1 on the day in question and the \( \text{UTB} \) argument lies in the range 0 to 1, or vice versa.

2. Both UT1 and TT are required, UT1 to predict the Earth rotation and TT to predict the effects of precession. If UT1 is used for both purposes, errors of order 100\( \mu \)as result.

3. This GAST is compatible with the IAU 2000/2006 resolutions and must be used only in conjunction with IAU 2006 precession and IAU 2000A mutation.

4. The result is returned in the range 0 to 2\( \pi \).

**REFERENCE:**

**iau\_GST94**  *Greenwich Apparent Sidereal Time, IAU 1982/94*  

**CALL:**

\[
D = \text{iau\_GST94} \ (UTA, \ \text{UTB})
\]

**ACTION:**


**GIVEN:**

\[
UTA, \ \text{UTB}
\]

UT1 as a two-part Julian Date (Notes 1,2)

**RETURNED** (function value):

\[
d
\]

Greenwich apparent sidereal time (radians)

**NOTES:**

1. The UT1 date \(UTA+UTB\) is a Julian Date, apportioned in any convenient way between the two arguments. For example, \(JD(UT1) = 2450123.7\) could be expressed in any of these ways, among others:

<table>
<thead>
<tr>
<th>UTA</th>
<th>UTB</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>2450123.7D0</td>
<td>0D0</td>
<td>(JD method)</td>
</tr>
<tr>
<td>2451545.0D0</td>
<td>−1421.3D0</td>
<td>(J2000 method)</td>
</tr>
<tr>
<td>2400000.5D0</td>
<td>50123.2D0</td>
<td>(MJD method)</td>
</tr>
<tr>
<td>2450123.5D0</td>
<td>0.2D0</td>
<td>(date &amp; time method)</td>
</tr>
</tbody>
</table>

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable (in the case of UT; the TT is not at all critical in this respect). The J2000 and MJD methods are good compromises between resolution and convenience. For UT, the date & time method is best matched to the algorithm that is used by the Earth Rotation Angle routine, called internally: maximum accuracy (or, at least, minimum noise) is delivered when the \(UTA\) argument is for 0h UT1 on the day in question and the \(UTB\) argument lies in the range 0 to 1, or *vice versa*.

2. The result is compatible with the IAU 1982 and 1994 resolutions, except that accuracy has been compromised for the sake of convenience in that UT is used instead of TDB (or TT) to compute the equation of the equinoxes.

3. This GAST must be used only in conjunction with contemporaneous IAU standards such as 1976 precession, 1980 obliquity and 1982 nutation. It is not compatible with the IAU 2000 resolutions.

4. The result is returned in the range 0 to \(2\pi\).

**REFERENCES:**

CALL :  
   CALL iau_IR ( R )

ACTION :  
   Initialize an r-matrix to the identity matrix.

RETURNED :  
   R  d(3,3)  r-matrix
CALL:
CALL iau_NUM00A ( DATE1, DATE2, RMATN )

ACTION:
Form the matrix of nutation for a given date, IAU 2000A model.

GIVEN:

| DATE1   | d   | TT as a two-part... |
| DATE2   | d   | ... Julian Date (Note 1) |

RETURNED:

| RMATN   | d(3,3) | nutation matrix N |

NOTES:

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

| DATE1   | DATE2       |          |
| 2450123.7D0 | 0D0 | JD method |
| 2451545.0D0 | −1421.3D0 | J2000 method |
| 2400000.5D0 | 50123.2D0 | MJD method |
| 2450123.5D0 | 0.2D0 | date & time method |

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The matrix operates in the sense

\[ \mathbf{v}_{\text{true}} = \mathbf{N} \mathbf{v}_{\text{mean}} , \]

where the p-vector \( \mathbf{v}_{\text{true}} \) is with respect to the true equatorial triad of date and the p-vector \( \mathbf{v}_{\text{mean}} \) is with respect to the mean equatorial triad of date.

3. A faster, but slightly less accurate, result (about 1 mas) can be obtained by using instead the iau_NUM00B routine.

REFERENCE:

CALL:

CALL iau_NUM00B ( DATE1, DATE2, RMATN )

ACTION:

Form the matrix of nutation for a given date, IAU 2000B model.

GIVEN:

\[
\begin{align*}
\text{DATE1} & \quad \text{d} \quad \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} \quad \ldots \text{Julian Date (Note 1)}
\end{align*}
\]

RETURNED:

\[
\begin{align*}
\text{RMATN} & \quad \text{d(3,3)} \quad \text{nutation matrix } N
\end{align*}
\]

NOTES:

1. The TT date \( \text{DATE1}+\text{DATE2} \) is a Julian Date, apportioned in any convenient way between the arguments \( \text{DATE1} \) and \( \text{DATE2} \). For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

\[
\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 \quad (\text{JD method}) \\
2451545.0D0 & \quad -1421.3D0 \quad (\text{J2000 method}) \\
2400000.5D0 & \quad 50123.2D0 \quad (\text{MJD method}) \\
2450123.5D0 & \quad 0.2D0 \quad (\text{date & time method})
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The matrix operates in the sense

\[
\mathbf{v}_{\text{true}} = \mathbf{N} \mathbf{v}_{\text{mean}},
\]

where the p-vector \( \mathbf{v}_{\text{true}} \) is with respect to the true equatorial triad of date and the p-vector \( \mathbf{v}_{\text{mean}} \) is with respect to the mean equatorial triad of date.

3. The present routine is faster, but slightly less accurate (about 1 mas), than the iau_NUM00A routine.

REFERENCE:

CALL:  
CALL iau_NUM06A ( DATE1, DATE2, RMATN )

ACTION:  
Form the matrix of nutation for a given date, IAU 2006/2000A model.

GIVEN:  
\begin{align*} 
\text{DATE1} & \quad \text{d} \quad \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} \quad \text{...Julian Date (Note 1)} 
\end{align*}

RETURNED:  
\begin{align*} 
\text{RMATN} & \quad \text{d(3,3)} \quad \text{nutation matrix N} 
\end{align*}

NOTES:  
1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:
\begin{align*} 
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 \quad \text{(JD method)} \\
2451545.0D0 & \quad -1421.3D0 \quad \text{(J2000 method)} \\
2400000.5D0 & \quad 50123.2D0 \quad \text{(MJD method)} \\
2450123.5D0 & \quad 0.2D0 \quad \text{(date & time method)} 
\end{align*}
The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The matrix operates in the sense
\[ \mathbf{v}_{\text{true}} = \mathbf{N} \mathbf{v}_{\text{mean}}, \]
where the p-vector \( \mathbf{v}_{\text{true}} \) is with respect to the true equatorial triad of date and the p-vector \( \mathbf{v}_{\text{mean}} \) is with respect to the mean equatorial triad of date.

REFERENCES:  
CALL:

CALL iau_NUMAT ( EPSA, DPSI, DEPS, RMATN )

ACTION:
Form the matrix of nutation, given $\Delta \psi$, $\Delta \epsilon$ and $\epsilon_A$.

GIVEN:

- $EPSA$ d mean obliquity of date $\epsilon_A$ (radians, Note 1)
- $DPSI$ d nutation in longitude $\Delta \psi$ (radians, Note 2)
- $DEPS$ d nutation in obliquity $\Delta \epsilon$ (radians, Note 2)

RETURNED:

- $RMATN$ d(3,3) nutation matrix (Note 3)

NOTES:

1. The supplied mean obliquity $EPSA$, must be consistent with the precession-nutation models from which $DPSI$ and $DEPS$ were obtained.

2. The caller is responsible for providing the nutation components; they are in longitude and obliquity, in radians and are with respect to the equinox and ecliptic of date.

3. The matrix operates in the sense

$$v_{\text{true}} = N v_{\text{mean}},$$

where the p-vector $v_{\text{true}}$ is with respect to the true equatorial triad of date and the p-vector $v_{\text{mean}}$ is with respect to the mean equatorial triad of date.

REFERENCE:

CALL: CALL iau_NUT00A ( DATE1, DATE2, DPSI, DEPS )


GIVEN: 

\[ \begin{align*} 
\text{DATE1} & \quad \text{d} \quad \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} \quad \ldots \text{Julian Date (Note 1)} 
\end{align*} \]

RETURNED: 

\[ \begin{align*} 
\text{DPSI} & \quad \text{d} \quad \text{nutation in longitude } \Delta \psi \text{ (radians, Note 2)} \\
\text{DEPS} & \quad \text{d} \quad \text{nutation in obliquity } \Delta \epsilon \text{ (radians, Note 2)} 
\end{align*} \]

NOTES:  

1. The TT date \text{DATE1}+\text{DATE2} is a Julian Date, apportioned in any convenient way between the arguments \text{DATE1} and \text{DATE2}. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

\[ \begin{align*} 
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 \quad \text{(JD method)} \\
2451545.0D0 & \quad -1421.3D0 \quad \text{(J2000 method)} \\
2400000.5D0 & \quad 50123.2D0 \quad \text{(MJD method)} \\
2450123.5D0 & \quad 0.2D0 \quad \text{(date & time method)} 
\end{align*} \]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The nutation components in longitude and obliquity are in radians and with respect to the equinox and ecliptic of date. The obliquity at J2000.0 is assumed to be the Lieske et al. (1977) value of 84381.448.

Both the luni-solar and planetary nutations are included. The latter are due to direct planetary nutations and the perturbations of the lunar and terrestrial orbits.

3. The routine computes the MHB2000 nutation series with the associated corrections for planetary nutations. It is an implementation of the nutation part of the IAU 2000A precession-nutation model, formally adopted by the IAU General Assembly in 2000, namely MHB2000 (Mathews et al. 2002), but with the free core nutation (FCN – see Note 4) omitted.
4. The full MHB2000 model also contains contributions to the nutations in longitude and obliquity due to the free-excitation of the free-core-nutation during the period 1979-2000. These FCN terms, which are time-dependent and unpredictable, are not included in the present routine and, if required, must be independently computed. With the FCN corrections included, the present routine delivers a pole which is at current epochs accurate to a few hundred microarcseconds. The omission of FCN introduces further errors of about that size.

5. The present routine provides classical nutation. The MHB2000 algorithm, from which it is adapted, deals also with (i) the offsets between the GCRS and mean poles and (ii) the adjustments in longitude and obliquity due to the changed precession rates. These additional topics, namely frame bias and precession adjustments, are supported by the SOFA routines iau_BI00 and iau_PR00.

6. The MHB2000 algorithm also provides “total” nutations, comprising the arithmetic sum of the frame bias, precession adjustments, luni-solar nutation and planetary nutation. These total nutations can be used in combination with an existing IAU 1976 precession implementation, such as iau_PMAT76, to deliver GCRS-to-true predictions of sub-mas accuracy at current epochs. However, there are three shortcomings in the MHB2000 model that must be taken into account if more accurate or definitive results are required (see Wallace 2002):

   (a) The MHB2000 total nutations are simply arithmetic sums, yet in reality the various components are successive Euler rotations. This slight lack of rigor leads to cross terms that exceed 1 mas after a century. The rigorous procedure is to form the GCRS-to-true rotation matrix by applying the bias, precession and nutation in that order.

   (b) Although the MHB2000 precession adjustments ($\Delta \psi, \Delta \epsilon$) are stated to be with respect to Lieske et al. (1977), the MHB2000 model does not specify which set of Euler angles are to be used and how the adjustments are to be applied. The most literal and straightforward procedure is to adopt the four-rotation

   $$ P = R_3(\chi_A) R_1(-\omega_A) R_3(-\psi_A) R_1(\epsilon_0) $$

   option, and to add $\Delta \psi$ to $\psi_A$ and $\Delta \epsilon$ to both $\omega_A$ and $\epsilon_A$.

   (c) The MHB2000 model predates the determination by Chapront et al. (2002) of a 14.6 mas displacement between the J2000.0 mean equinox and the origin of the ICRS frame. It should, however, be noted that neglecting this displacement when calculating star coordinates does not lead to a 14.6 mas change in right ascension, only a small second-order distortion in the pattern of the precession-nutation effect.

   For these reasons, the SOFA routines do not generate the “total nutations” directly, though they can of course easily be generated by calling iau_BI00, iau_PR00 and the present routine and adding the results.

7. The MHB2000 model contains 41 instances where the same frequency appears multiple times, of which 38 are duplicates and three are triplicates. To keep the present code close to the original MHB algorithm, this small inefficiency has not been corrected.
REFERENCES:

CALL:  
CALL iau_NUT00B ( DATE1, DATE2, DPSI, DEPS )

ACTION:  
Nutation, IAU 2000B model.

GIVEN:  

\begin{align*} 
\text{DATE1} & \quad \text{d} \quad \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} \quad \text{...Julian Date (Note 1)}
\end{align*}

RETURNED:  

\begin{align*} 
\text{DPSI} & \quad \text{d} \quad \text{nutation in longitude } \Delta \psi \text{ (radians, Note 2)} \\
\text{DEPS} & \quad \text{d} \quad \text{nutation in obliquity } \Delta \epsilon \text{ (radians, Note 2)}
\end{align*}

NOTES:  

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

\begin{align*} 
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 \quad \text{(JD method)} \\
2451545.0D0 & \quad -1421.3D0 \quad \text{(J2000 method)} \\
2400000.5D0 & \quad 50123.2D0 \quad \text{(MJD method)} \\
2450123.5D0 & \quad 0.2D0 \quad \text{(date & time method)}
\end{align*}

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The nutation components in longitude and obliquity are in radians and with respect to the equinox and ecliptic of date. The obliquity at J2000.0 is assumed to be the Lieske et al. (1977) value of 84381.448. (The errors that result from using this routine with the IAU 2006 value of 84381.406 can be neglected.)

The nutation model consists only of luni-solar terms, but includes also a fixed offset which compensates for certain long-period planetary terms (Note 7).

3. This routine is an implementation of the IAU 2000B abridged nutation model formally adopted by the IAU General Assembly in 2000. The routine computes the MHB_2000_SHORT luni-solar nutation series (Luzum 2001), but without the associated corrections for the precession rate adjustments and the offset between the GCRS and J2000.0 mean poles.

The routine delivers a pole accurate to 1 mas from 1900 to 2100 (usually better than 1 mas, very occasionally just outside 1 mas). The full IAU 2000A model, which is implemented in the routine iau_NUT00A (q.v.), delivers considerably greater accuracy at current epochs; however, to realize this improved accuracy, corrections for the essentially unpredictable free-core-nutation (FCN) must also be included.

5. The present routine provides classical nutation. The MHB2000 algorithm, from which it is adapted, deals also with (i) the offsets between the GCRS and mean poles and (ii) the adjustments in longitude and obliquity due to the changed precession rates. These additional topics, namely frame bias and precession adjustments, are supported by the SOFA routines iau_BI00 and iau_PR00.

6. The MHB_2000_SHORT algorithm also provides “total” nutations, comprising the arithmetic sum of the frame bias, precession adjustments, and nutation (luni-solar + planetary). These total nutations can be used in combination with an existing IAU 1976 precession implementation, such as iau_PMAT76, to deliver GCRS-to-true predictions of milliarcsecond accuracy at current epochs. However, for symmetry with the iau_NUT00A routine (q.v. for the reasons), the SOFA routines do not generate the “total nutations” directly. Should they be required, they could of course easily be generated by calling iau_BI00, iau_PR00 and the present routine and adding the results.

7. The IAU 2000B model includes “planetary bias” terms that are fixed in size but compensate for long-period nutations. The amplitudes quoted in McCarthy & Luzum (2003), namely $\Delta \psi = -1.5835$ mas and $\Delta \epsilon = +1.6339$ mas, are optimized for the “total nutations” method described in Note 6. The Luzum (2001) values used in this SOFA implementation, namely $-0.135$ mas and $+0.388$ mas, are optimized for the “rigorous” method, where frame bias, precession and nutation are applied separately and in that order. During the interval 1995-2050, the SOFA implementation delivers a maximum error of 1.001 mas (not including FCN).

REFERENCES:

2. Luzum, B., private communication, 2001 (Fortran code MHB_2000_SHORT).
CALL:
CALL iau_NUT06A ( DATE1, DATE2, DPSI, DEPS )

ACTION:
IAU 2000A nutation with adjustments to match the IAU 2006 precession.

GIVEN:

DATE1  d  TT as a two-part...
DATE2  d  …Julian Date (Note 1)

RETURNED:

DPSI  d  nutation in longitude $\Delta \psi$ (radians, Note 2)
DEPS  d  nutation in obliquity $\Delta \epsilon$ (radians, Note 2)

NOTES:

1. The TT date $DATE1+DATE2$ is a Julian Date, apportioned in any convenient way between the arguments $DATE1$ and $DATE2$. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

<table>
<thead>
<tr>
<th>DATE1</th>
<th>DATE2</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2450123.7D0</td>
<td>0D0</td>
<td>(JD method)</td>
</tr>
<tr>
<td>2451545.0D0</td>
<td>−1421.3D0</td>
<td>(J2000 method)</td>
</tr>
<tr>
<td>2400000.5D0</td>
<td>50123.2D0</td>
<td>(MJD method)</td>
</tr>
<tr>
<td>2450123.5D0</td>
<td>0.2D0</td>
<td>(date &amp; time method)</td>
</tr>
</tbody>
</table>

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The nutation components in longitude and obliquity are in radians and with respect to the mean equinox and ecliptic of date, IAU 2006 precession model (Hilton et al., 2006, Capitaine et al., 2005)

3. The routine first computes the IAU 2000A nutation, then applies adjustments for (i) the consequences of the change in obliquity from the IAU 1980 ecliptic to the IAU 2006 ecliptic and (ii) the secular variation in the Earth’s dynamical form factor $J_2$.

4. The present routine provides classical nutation, complementing the IAU 2000 frame bias and IAU 2006 precession. It delivers a pole which is at current epochs accurate to a few tens of microarcseconds, apart from the free core nutation.
REFERENCES:

CALL:
CALL iau_NUT80 ( DATE1, DATE2, DPSI, DEPS )

ACTION:
Nutation, IAU 1980 model.

GIVEN:

\[
\begin{align*}
\text{DATE1} & \quad \text{d} & & \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} & & \text{...Julian Date (Note 1)}
\end{align*}
\]

RETURNED:

\[
\begin{align*}
\text{DPSI} & \quad \text{d} & & \text{nutation in longitude } \Delta \psi \text{ (radians, Note 2)} \\
\text{DEPS} & \quad \text{d} & & \text{nutation in obliquity } \Delta \varepsilon \text{ (radians, Note 2)}
\end{align*}
\]

NOTES:

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

\[
\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
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2450123.5D0 & \quad 0.2D0 & \text{(date & time method)}
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The nutation components are with respect to the ecliptic of date.

REFERENCE:

**CALL:**

CALL iau_NUTM80 ( DATE1, DATE2, RMATN )

**ACTION:**

Form the matrix of nutation for a given date, IAU 1980 model.

**GIVEN:**

\[
\begin{align*}
\text{DATE1} & \quad d & \text{TT as a two-part...} \\
\text{DATE2} & \quad d & \ldots \text{Julian Date (Note 1)} \\
\end{align*}
\]

**RETURNED:**

\[
\begin{align*}
\text{RMATN} & \quad d(3,3) & \text{nutation matrix N} \\
\end{align*}
\]

**NOTES:**

1. The TT date \( \text{DATE1+DATE2} \) is a Julian Date, apportioned in any convenient way between the arguments \( \text{DATE1} \) and \( \text{DATE2} \). For example, \( \text{JD(TT)} = 2450123.7 \) could be expressed in any of these ways, among others:

\[
\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 & \text{(JD method)} \\
2451545.0D0 & \quad -1421.3D0 & \text{(J2000 method)} \\
2400000.5D0 & \quad 50123.2D0 & \text{(MJD method)} \\
2450123.5D0 & \quad 0.2D0 & \text{(date & time method)} \\
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The matrix operates in the sense

\[
\mathbf{v}_{\text{true}} = \mathbf{N} \mathbf{v}_{\text{mean}},
\]

where the p-vector \( \mathbf{v}_{\text{true}} \) is with respect to the true equatorial triad of date and the p-vector \( \mathbf{v}_{\text{mean}} \) is with respect to the mean equatorial triad of date.
CALL:
\[ D = \text{iau\_OBL06}(\text{DATE1}, \text{DATE2}) \]

ACTION:
Mean obliquity of the ecliptic, IAU 2006 precession model.

GIVEN:
\[ \text{DATE1} \quad \text{d} \quad \text{TT as a two-part...} \]
\[ \text{DATE2} \quad \text{d} \quad \text{...Julian Date (Note 1)} \]

RETURNED (function value):
\[ d \quad \text{obliquity of the ecliptic (radians, Note 2)} \]

NOTES:
1. The TT date \( \text{DATE1+DATE2} \) is a Julian Date, apportioned in any convenient way between the arguments \( \text{DATE1} \) and \( \text{DATE2} \). For example, \( \text{JD(TT)} = 2450123.7 \) could be expressed in any of these ways, among others:

\[
\begin{array}{c|c|c}
\text{DATE1} & \text{DATE2} & \text{Note} \\
2450123.7D0 & 0D0 & \text{(JD method)} \\
2451545.0D0 & -1421.3D0 & \text{(J2000 method)} \\
2400000.5D0 & 50123.2D0 & \text{(MJD method)} \\
2450123.5D0 & 0.2D0 & \text{(date & time method)} \\
\end{array}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The result is the angle between the ecliptic and mean equator of date \( \text{DATE1+DATE2} \).

REFERENCE:
CALL:
\[ D = \text{iau}\_\text{OBL80}(\text{DATE1}, \text{DATE2}) \]

ACTION:
Mean obliquity of the ecliptic, IAU 1980 precession model.

GIVEN:
- \text{DATE1}: d TT as a two-part...
- \text{DATE2}: d ...Julian Date (Note 1)

RETURNED (function value):
- d obliquity of the ecliptic (radians, Note 2)

NOTES:

1. The TT date \text{DATE1+DATE2} is a Julian Date, apportioned in any convenient way between the arguments \text{DATE1} and \text{DATE2}. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:
   \begin{align*}
   \text{DATE1} & \quad \text{DATE2} \\
   2450123.7D0 & \quad 0D0 \quad \text{(JD method)} \\
   2451545.0D0 & \quad −1421.3D0 \quad \text{(J2000 method)} \\
   2400000.5D0 & \quad 50123.2D0 \quad \text{(MJD method)} \\
   2450123.5D0 & \quad 0.2D0 \quad \text{(date & time method)}
   \end{align*}

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The result is the angle between the ecliptic and mean equator of date \text{DATE1+DATE2}.

REFERENCE:
CALL:  
CALL iau_P06E ( DATE1, DATE2, EPS0, PSIA, OMA, BPA, BQA, PIA, BPIA, EPSA, CHIA, ZA, ZETAA, THETAA, PA, GAM, PHI, PSI )

ACTION:  
Precession angles, IAU 2006, equinox based.

GIVEN:  

\[
\begin{align*}
\text{DATE1} & \quad \text{d} & \quad \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} & \quad \text{...Julian Date (Note 1)}
\end{align*}
\]

RETURNED (see Note 2):  

\[
\begin{align*}
\text{EPS0} & \quad \text{d} & \quad \epsilon_0 \\
\text{PSIA} & \quad \text{d} & \quad \psi_A \\
\text{OMA} & \quad \text{d} & \quad \omega_A \\
\text{BPA} & \quad \text{d} & \quad P_A \\
\text{BQA} & \quad \text{d} & \quad Q_A \\
\text{PIA} & \quad \text{d} & \quad \pi_A \\
\text{BPIA} & \quad \text{d} & \quad \Pi_A \\
\text{EPSA} & \quad \text{d} & \quad \epsilon_A \\
\text{CHIA} & \quad \text{d} & \quad \chi_A \\
\text{ZA} & \quad \text{d} & \quad z_A \\
\text{ZETAA} & \quad \text{d} & \quad \zeta_A \\
\text{THETAA} & \quad \text{d} & \quad \theta_A \\
\text{PA} & \quad \text{d} & \quad p_A \\
\text{GAM} & \quad \text{d} & \quad \gamma_{J2000} \\
\text{PHI} & \quad \text{d} & \quad \phi_{J2000} \\
\text{PSI} & \quad \text{d} & \quad \psi_{J2000}
\end{align*}
\]

NOTES:  

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

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\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
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2450123.5D0 & \quad 0.2D0 & \quad \text{(date & time method)}
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.
2. This routine returns the set of equinox based angles for the Capitaine et al. “P03” precession theory, adopted by the IAU in 2006. The angles are set out in Table 1 of Hilton et al. (2006):

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPS0</td>
<td>$\epsilon_0$ obliquity at J2000.0</td>
</tr>
<tr>
<td>PSI A</td>
<td>$\psi_A$ luni-solar precession</td>
</tr>
<tr>
<td>OMA</td>
<td>$\omega_A$ inclination of equator wrt J2000.0 ecliptic</td>
</tr>
<tr>
<td>BPA</td>
<td>$P_A$ ecliptic pole $x$, J2000.0 ecliptic triad</td>
</tr>
<tr>
<td>BQA</td>
<td>$Q_A$ ecliptic pole $-y$, J2000.0 ecliptic triad</td>
</tr>
<tr>
<td>PIA</td>
<td>$\pi_A$ angle between moving and J2000.0 ecliptics</td>
</tr>
<tr>
<td>BPI A</td>
<td>$\Pi_A$ longitude of ascending node of the ecliptic</td>
</tr>
<tr>
<td>EPSA</td>
<td>$\epsilon_A$ obliquity of the ecliptic</td>
</tr>
<tr>
<td>CHIA</td>
<td>$\chi_A$ planetary precession</td>
</tr>
<tr>
<td>ZA</td>
<td>$z_A$ equatorial precession: $-3$rd 323 Euler angle</td>
</tr>
<tr>
<td>ZETAA</td>
<td>$\zeta_A$ equatorial precession: $-1$st 323 Euler angle</td>
</tr>
<tr>
<td>THETAA</td>
<td>$\theta_A$ equatorial precession: $2$nd 323 Euler angle</td>
</tr>
<tr>
<td>PA</td>
<td>$p_A$ general precession (n.b. see below)</td>
</tr>
<tr>
<td>GAM</td>
<td>$\gamma_{J2000}$ J2000.0 RA difference of ecliptic poles</td>
</tr>
<tr>
<td>PHI</td>
<td>$\phi_{J2000}$ J2000.0 codeclination of ecliptic pole</td>
</tr>
<tr>
<td>PSI</td>
<td>$\psi_{J2000}$ longitude difference of equator poles, J2000.0</td>
</tr>
</tbody>
</table>

The returned values are all radians.

Note that the $t^5$ coefficient in the series for $p_A$ from Capitaine et al. (2003) is incorrectly signed in Hilton et al. (2006).

3. Hilton et al. (2006) Table 1 also contains angles that depend on models distinct from the P03 precession theory itself, namely the IAU 2000A frame bias and nutation. The quoted polynomials are used in other SOFA routines:

- iau_XY06 contains the polynomial parts of the $X$ and $Y$ series.
- iau_S06 contains the polynomial part of the $s + XY/2$ series.
- iau_PFW06 implements the series for the Fukushima-Williams angles that are with respect to the GCRS pole (i.e. the variants that include frame bias).

4. The IAU resolution stipulated that the choice of parameterization was left to the user, and so an IAU compliant precession implementation can be constructed using various combinations of the angles returned by the present routine.

5. The parameterization used by SOFA is the version of the Fukushima-Williams angles that refers directly to the GCRS pole. These angles may be calculated by calling the routine iau_PFW06. SOFA also supports the direct computation of the CIP GCRS $X,Y$ by series, available by calling iau_XY06.

6. The agreement between the different parameterizations is at the $1\mu$as level in the present era.

7. When constructing a precession formulation that refers to the GCRS pole rather than the dynamical pole, it may (depending on the choice of angles) be necessary to introduce the frame bias explicitly.

REFERENCES:

CALL:
CALL iau_PB06 ( DATE1, DATE2, BZETA, BZ, BTHETA )

ACTION:
This routine forms three Euler angles which implement general precession from epoch J2000.0, using the IAU 2006 model. Frame bias (the offset between ICRS and mean J2000.0) is included.

 GIVEN:
 DATE1 d TT as a two-part...
 DATE2 d ...Julian Date (Note 1)

 RETURNED:
 BZETA d 1st rotation ζ: radians clockwise around z
 BZ d 3rd rotation z: radians clockwise around z
 BTHETA d 2nd rotation θ: radians counterclockwise around y

 NOTES:
1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

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The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The traditional accumulated precession angles ζ_{A}, z_{A} and θ_{A} cannot be obtained in the usual way, namely through polynomial expressions, because of the frame bias. The latter means that two of the angles undergo rapid changes near this date. They are instead the results of decomposing the precession-bias matrix obtained by using the Fukushima-Williams method, which does not suffer from the problem. The decomposition returns values which can be used in the conventional formulation and which include frame bias.

3. The three angles are returned in the conventional order, which is not the same as the order of the corresponding Euler rotations. The precession-bias matrix (GCRS to CIRS) is $R_3(\zeta) R_2(\theta) R_3(-\zeta)$.

4. Should ζ_{A}, z_{A}, θ_{A} angles be required that do not contain frame bias, they are available by calling the SOFA routine iau_P06E.
CALL:
CALL iau_PFW06 ( DATE1, DATE2, GAMB, PHIB, PSIB, EPSA )

ACTION:
Precession angles, IAU 2006, Fukushima-Williams 4-angle formulation.

GIVEN:

\begin{align*}
\text{DATE1} & \quad \text{d} \quad \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} \quad \ldots\text{Julian Date (Note 1)}
\end{align*}

RETURNED:

\begin{align*}
\text{GAMB} & \quad \text{d} \quad \text{F-W angle } \bar{\gamma} \text{ (radians)} \\
\text{PHIB} & \quad \text{d} \quad \text{F-W angle } \bar{\phi} \text{ (radians)} \\
\text{PSIB} & \quad \text{d} \quad \text{F-W angle } \bar{\psi} \text{ (radians)} \\
\text{EPSA} & \quad \text{d} \quad \text{F-W angle } \epsilon_A \text{ (radians)}
\end{align*}

NOTES:

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

\begin{align*}
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2450123.5D0 & \quad 0.2D0 \quad \text{(date \& time method)}
\end{align*}

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date \& time methods are both good compromises between resolution and convenience.

2. Naming the following points:

- \( e = \text{J2000.0 ecliptic pole} \)
- \( p = \text{GCRS pole} \)
- \( E = \text{ecliptic pole of date} \)
- \( P = \text{mean pole of date} \)

the four Fukushima-Williams angles are as follows:

\begin{align*}
\bar{\gamma} & = e\bar{E} \\
\bar{\phi} & = p\bar{E} \\
\bar{\psi} & = p\bar{P} \\
\epsilon_A & = \bar{E}
\end{align*}
3. The matrix representing the combined effects of frame bias and precession is:

$$P B = R_1(-\epsilon_A) R_3(-\bar{\psi}) R_1(\bar{\phi}) R_3(\bar{\gamma})$$

4. The matrix representing the combined effects of frame bias, precession and nutation is:

$$NPB = R_1(-\epsilon_A + \delta\epsilon) R_3(-\bar{\psi} + \delta\psi) R_1(\bar{\phi}) R_3(\bar{\gamma})$$

where $\delta\psi$ and $\delta\epsilon$ are the nutation components with respect to the ecliptic of date.

REFERENCE:

CALL:  
CALL iau_PMAT00 ( DATE1, DATE2, RBP )

ACTION:  
Precession matrix (including frame bias) from GCRS to a specified date, IAU 2000 model.

GIVEN:  

\[
\begin{align*}
DATE1 & \quad d \quad \text{TT as a two-part...} \\
DATE2 & \quad d \quad \ldots \text{Julian Date (Note 1)}
\end{align*}
\]

RETURNED:  

\[
\begin{align*}
RBP & \quad d(3,3) \quad \text{bias-precession matrix } \mathbf{BP} \text{ (Note 2)}
\end{align*}
\]

NOTES:  

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

\[
\begin{align*}
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2450123.5D0 & \quad 0.2D0 \quad \text{(date & time method)}
\end{align*}
\]

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2. The matrix operates in the sense

\[
\mathbf{v}_{\text{date}} = \mathbf{BP} \mathbf{v}_{\text{GCRS}}
\]

where the p-vector \(\mathbf{v}_{\text{date}}\) is with respect to the mean equatorial triad of the given date and the p-vector \(\mathbf{v}_{\text{GCRS}}\) is with respect to the Geocentric Celestial Reference System (IAU, 2000).

REFERENCE:  

CALL:
CALL iau_PMAT06 ( DATE1, DATE2, RBP )

ACTION:
Precession matrix (including frame bias) from GCRS to a specified date, IAU 2006 model.

GIVEN:

\[
\begin{align*}
\text{DATE1} & \quad \text{d} & \quad \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} & \quad \ldots \text{Julian Date (Note 1)} \\
\end{align*}
\]

RETURNED:

\[
\begin{align*}
\text{RBP} & \quad \text{d(3,3)} & \quad \text{bias-precession matrix BP (Note 2)} \\
\end{align*}
\]

NOTES:

1. The TT date \(\text{DATE1}+\text{DATE2}\) is a Julian Date, apportioned in any convenient way between the arguments \(\text{DATE1}\) and \(\text{DATE2}\). For example, \(\text{JD(TT)} = 2450123.7\) could be expressed in any of these ways, among others:

\[
\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 & \quad \text{(JD method)} \\
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2400000.5D0 & \quad 50123.2D0 & \quad \text{(MJD method)} \\
2450123.5D0 & \quad 0.2D0 & \quad \text{(date & time method)} \\
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The matrix operates in the sense

\[
\mathbf{v}_{\text{date}} = \mathbf{BP} \mathbf{v}_{\text{GCRS}} ,
\]

where the p-vector \(\mathbf{v}_{\text{date}}\) is with respect to the mean equatorial triad of the given date and the p-vector \(\mathbf{v}_{\text{GCRS}}\) is with respect to the Geocentric Celestial Reference System (IAU, 2000).

REFERENCES:

CALL : 
CALL iau_PMAT76 ( DATE1, DATE2, RMATP )

ACTION : 
Precession matrix from J2000.0 to a specified date, IAU 1976 model.

GIVEN : 
DATE1    d
         TT as a two-part... 
DATE2    d
         ...Julian Date (Note 1)

RETURNED : 
RMATP    d(3,3) 
precession matrix \( P \) (Note 2)

NOTES : 
1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way 
   between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could 
   be expressed in any of these ways, among others:

   \[
   \begin{align*}
   &\text{DATE1} & \text{DATE2} \\
   &2450123.7D0 & 0D0 & \text{(JD method)} \\
   &2451545.0D0 & -1421.3D0 & \text{(J2000 method)} \\
   &2400000.5D0 & 50123.2D0 & \text{(MJD method)} \\
   &2450123.5D0 & 0.2D0 & \text{(date & time method)}
   \end{align*}
   \]

   The JD method is the most natural and convenient to use in cases where the loss of 
   several decimal digits of resolution is acceptable. The J2000 method is best matched 
   to the way the argument is handled internally and will deliver the optimum resolution. 
   The MJD method and the date & time methods are both good compromises between 
   resolution and convenience.

2. The matrix operates in the sense 
   \[ v_{\text{date}} = P v_{\text{J2000}} , \]
   where the p-vector \( v_{\text{date}} \) is with respect to the mean equatorial triad of the given 
   date and the p-vector \( v_{\text{J2000}} \) is with respect to the mean equatorial triad of the given 
   date.

3. Though the matrix method itself is rigorous, the precession angles are expressed 
   through canonical polynomials which are valid only for a limited time span. In 
   addition, the IAU 1976 precession rate is known to be imperfect. The absolute 
   accuracy of the present formulation is better than 0''1 from 1960 CE to 2040 CE, 
   better than 1'' from 1640 CE to 2360 CE, and remains below 3'' for the whole of 
   the period 500 CE to 3000 CE. The errors exceed 10'' outside the range 1200 CE to 
   3900 CE, exceed 100'' outside 4200 CE to 5600 CE and exceed 1000'' outside 6800 CE 
   to 8200 CE.

REFERENCES : 
CALL:

CALL iau_PN00 ( DATE1, DATE2, DPSI, DEPS, EPSA, RB, RP, RBP, RN, RBPN )

ACTION:


GIVEN:

DATE1 d TT as a two-part...
DATE2 d ...Julian Date (Note 1)
DPSI d nutation in longitude $\Delta \psi$ (radians, Note 2)
DEPS d nutation in obliquity $\Delta \epsilon$ (radians, Note 2)

RETURNED:

EPSA d mean obliquity $\epsilon_A$ (Note 3)
RB d(3,3) frame bias matrix B (Note 4)
RP d(3,3) precession matrix P (Note 5)
RBP d(3,3) bias-precession matrix BP (Note 6)
RN d(3,3) nutation matrix N (Note 7)
RBPN d(3,3) GCRS-to-true matrix BPN (Notes 8,9)

NOTES:

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

<table>
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<tr>
<th>DATE1</th>
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</thead>
<tbody>
<tr>
<td>2450123.7D0</td>
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<td>(JD method)</td>
</tr>
<tr>
<td>2451545.0D0</td>
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</tr>
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The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The caller is responsible for providing the nutation components; they are in longitude and obliquity, in radians and are with respect to the equinox and ecliptic of date. For high-accuracy applications, free core nutation should be included as well as any other relevant corrections to the position of the CIP.
3. The returned mean obliquity $\epsilon_A$ is consistent with the IAU 2000 precession-nutation models.

4. The matrix $\mathbf{B}$ transforms vectors from GCRS to J2000.0 mean equator and equinox by applying frame bias.

5. The matrix $\mathbf{P}$ transforms vectors from J2000.0 mean equator and equinox to mean equator and equinox of date by applying precession.

6. The matrix $\mathbf{BP}$ transforms vectors from GCRS to mean equator and equinox of date by applying frame bias then precession. It is the product $\mathbf{PB}$.

7. The matrix $\mathbf{N}$ transforms vectors from mean equator and equinox of date to true equator and equinox of date by applying the nutation (luni-solar + planetary).

8. The matrix $\mathbf{BPN}$ transforms vectors from GCRS to true equator and equinox of date. It is the product $\mathbf{NPB}$, applying frame bias, precession and nutation in that order.

**REFERENCE:**

CALL:

CALL iau_PN00A ( DATE1, DATE2, DPSI, DEPS, EPSA, RB, RP, RBP, RN, RBPN )

ACTION:


GIVEN:

- DATE1: d TT as a two-part...
- DATE2: d ...Julian Date (Note 1)

RETURNED:

- DPSI: d mutation in longitude $\Delta \psi$ (radians, Note 2)
- DEPS: d mutation in obliquity $\Delta \epsilon$ (radians, Note 2)
- EPSA: d mean obliquity $\epsilon_A$ (Note 3)
- RB: d(3,3) frame bias matrix $B$ (Note 4)
- RP: d(3,3) precession matrix $P$ (Note 5)
- RBP: d(3,3) bias-precession matrix $BP$ (Note 6)
- RN: d(3,3) nutation matrix $N$ (Note 7)
- RBPN: d(3,3) GCRS-to-true matrix $BPN$ (Notes 8,9)

NOTES:

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

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<td>-1421.3D0</td>
</tr>
<tr>
<td>2400000.5D0</td>
<td>50123.2D0</td>
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   The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The returned nutation components $\Delta \psi, \Delta \epsilon$ (luni-solar + planetary, IAU 2000A) are in radians and with respect to the equinox and ecliptic of date. Free core nutation is omitted; for the utmost accuracy, use the iau_PN00 routine, where the nutation components are caller-specified. For faster but slightly less accurate results, use the iau_PN00B routine.
3. The returned mean obliquity $\epsilon_A$ is consistent with the IAU 2000 precession.

4. The matrix $B$ transforms vectors from GCRS to J2000.0 mean equator and equinox by applying frame bias.

5. The matrix $P$ transforms vectors from J2000.0 mean equator and equinox to mean equator and equinox of date by applying precession.

6. The matrix $BP$ transforms vectors from GCRS to mean equator and equinox of date by applying frame bias then precession. It is the product $PB$.

7. The matrix $N$ transforms vectors from mean equator and equinox of date to true equator and equinox of date by applying the nutation (luni-solar + planetary).

8. The matrix $BPN$ transforms vectors from GCRS to true equator and equinox of date. It is the product $NPB$, applying frame bias, precession and nutation in that order.


**REFERENCE**

CALL:
CALL iau_PN00B ( DATE1, DATE2, DPSI, DEPS, EPSA, RB, RP, RBP, RN, RBPN )

ACTION:

GIVEN:
DATE1 d TT as a two-part...
DATE2 d ...Julian Date (Note 1)

RETURNED:
DPSI d nutation in longitude \( \Delta \psi \) (radians, Note 2)
DEPS d nutation in obliquity \( \Delta \epsilon \) (radians, Note 2)
EPSA d mean obliquity \( \epsilon_A \) (Note 3)
RB d(3,3) frame bias matrix \( B \) (Note 4)
RP d(3,3) precession matrix \( P \) (Note 5)
RBP d(3,3) bias-precession matrix \( BP \) (Note 6)
RN d(3,3) nutation matrix \( N \) (Note 7)
RBPN d(3,3) GCRS-to-true matrix \( BPN \) (Notes 8,9)

NOTES:
1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

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<td>(J2000 method)</td>
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The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The returned nutation components \( \Delta \psi, \Delta \epsilon \) (luni-solar + planetary, IAU 2000B) are in radians and with respect to the equinox and ecliptic of date. For more accurate results, but at the cost of increased computation, use the iau_PN00A routine. For the utmost accuracy, use the iau_PN00 routine, where the nutation components are caller-specified.
3. The returned mean obliquity $\epsilon_A$ is consistent with the IAU 2000 precession.

4. The matrix $B$ transforms vectors from GCRS to J2000.0 mean equator and equinox by applying frame bias.

5. The matrix $P$ transforms vectors from J2000.0 mean equator and equinox to mean equator and equinox of date by applying precession.

6. The matrix $BP$ transforms vectors from GCRS to mean equator and equinox of date by applying frame bias then precession. It is the product $PB$.

7. The matrix $N$ transforms vectors from mean equator and equinox of date to true equator and equinox of date by applying the nutation (luni-solar + planetary).

8. The matrix $BPN$ transforms vectors from GCRS to true equator and equinox of date. It is the product $NPB$, applying frame bias, precession and nutation in that order.


**REFERENCE:**

CALL:

CALL iau_PN06 ( DATE1, DATE2, DPSI, DEPS, EPSA, RB, RP, RBP, RN, RBPN )

ACTION:

Precession-nutation, IAU 2006 model: a multi-purpose routine, supporting classical (equinox-based) use directly and CIO-based use indirectly.

GIVEN:

\begin{align*}
\text{DATE1} & \quad \text{d} \quad \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} \quad \ldots \text{Julian Date (Note 1)} \\
\text{DPSI} & \quad \text{d} \quad \text{nutation in longitude } \Delta \psi \text{ (radians, Note 2)} \\
\text{DEPS} & \quad \text{d} \quad \text{nutation in obliquity } \Delta \epsilon \text{ (radians, Note 2)} \\
\end{align*}

RETURNED:

\begin{align*}
\text{EPSA} & \quad \text{d} \quad \text{mean obliquity } \epsilon_A \text{ (Note 3)} \\
\text{RB} & \quad \text{d(3,3)} \quad \text{frame bias matrix } B \text{ (Note 4)} \\
\text{RP} & \quad \text{d(3,3)} \quad \text{precession matrix } P \text{ (Note 5)} \\
\text{RBP} & \quad \text{d(3,3)} \quad \text{bias-precession matrix } BP \text{ (Note 6)} \\
\text{RN} & \quad \text{d(3,3)} \quad \text{nutation matrix } N \text{ (Note 7)} \\
\text{RBPN} & \quad \text{d(3,3)} \quad \text{GCRS-to-true matrix } BPN \text{ (Notes 8,9)} \\
\end{align*}

NOTES:

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
2450123.70 & \quad 00 \quad \text{(JD method)} \\
2451545.09 & \quad -1421.30 \quad \text{(J2000 method)} \\
2400000.50 & \quad 50123.20 \quad \text{(MJD method)} \\
2450123.50 & \quad 0.20 \quad \text{(date & time method)} \\
\end{align*}

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The caller is responsible for providing the nutation components; they are in longitude and obliquity, in radians and are with respect to the equinox and ecliptic of date. For high-accuracy applications, free core nutation should be included as well as any other relevant corrections to the position of the CIP.

3. The returned mean obliquity $\epsilon_A$ is consistent with the IAU 2006 precession.
4. The matrix $B$ transforms vectors from GCRS to J2000.0 mean equator and equinox by applying frame bias.

5. The matrix $P$ transforms vectors from J2000.0 mean equator and equinox to mean equator and equinox of date by applying precession.

6. The matrix $BP$ transforms vectors from GCRS to mean equator and equinox of date by applying frame bias then precession. It is the product $PB$.

7. The matrix $N$ transforms vectors from mean equator and equinox of date to true equator and equinox of date by applying the nutation (luni-solar + planetary).

8. The matrix $BPN$ transforms vectors from GCRS to true equator and equinox of date. It is the product $NPB$, applying frame bias, precession and nutation in that order.

9. The $X,Y,Z$ coordinates of the IAU 2006 Celestial Intermediate Pole are elements (3,1-3) of the GCRS-to-true matrix $BPN$.

REFERENCE:

CALL:  
CALL iau_PN06A ( DATE1, DATE2, DPSI, DEPS, EPSA, RB, RP, RBP, RN, RBPN )

ACTION:  

GIVEN:  

\begin{itemize}
  \item \texttt{DATE1} \hspace{1cm} \texttt{d} \hspace{1cm} TT as a two-part...
  \item \texttt{DATE2} \hspace{1cm} \texttt{d} \hspace{1cm} \ldots Julian Date (Note 1)
\end{itemize}

RETURNED:  

\begin{itemize}
  \item \texttt{DPSI} \hspace{1cm} \texttt{d} \hspace{1cm} nutation in longitude $\Delta \psi$ (radians, Note 2)
  \item \texttt{DEPS} \hspace{1cm} \texttt{d} \hspace{1cm} nutation in obliquity $\Delta \epsilon$ (radians, Note 2)
  \item \texttt{EPSA} \hspace{1cm} \texttt{d} \hspace{1cm} mean obliquity $\epsilon_A$ (Note 3)
  \item \texttt{RB} \hspace{1cm} \texttt{d(3,3)} \hspace{1cm} frame bias matrix $B$ (Note 4)
  \item \texttt{RP} \hspace{1cm} \texttt{d(3,3)} \hspace{1cm} precession matrix $P$ (Note 5)
  \item \texttt{RBP} \hspace{1cm} \texttt{d(3,3)} \hspace{1cm} bias-precession matrix $BP$ (Note 6)
  \item \texttt{RN} \hspace{1cm} \texttt{d(3,3)} \hspace{1cm} nutation matrix $N$ (Note 7)
  \item \texttt{RBPN} \hspace{1cm} \texttt{d(3,3)} \hspace{1cm} GCRS-to-true matrix $BPN$ (Notes 8,9)
\end{itemize}

NOTES:  

1. The TT date \texttt{DATE1+DATE2} is a Julian Date, apportioned in any convenient way between the arguments \texttt{DATE1} and \texttt{DATE2}. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

\begin{center}
\begin{tabular}{ccc}
\texttt{DATE1} & \texttt{DATE2} & \texttt{JD method} \\
2450123.7D0 & 0D0 & \\
2451545.0D0 & -1421.3D0 & (J2000 method) \\
2400000.5D0 & 50123.2D0 & (MJD method) \\
2450123.5D0 & 0.2D0 & (date & time method)
\end{tabular}
\end{center}

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The returned nutation components $\Delta \psi, \Delta \epsilon$ (luni-solar + planetary, IAU 2006A) are in radians and with respect to the equinox and ecliptic of date. Free core nutation is omitted; for the utmost accuracy, use the \texttt{iau_PN06} routine, where the nutation components are caller-specified.

3. The returned mean obliquity $\epsilon_A$ is consistent with the IAU 2000 precession.
4. The matrix $\mathbf{B}$ transforms vectors from GCRS to J2000.0 mean equator and equinox by applying frame bias.

5. The matrix $\mathbf{P}$ transforms vectors from J2000.0 mean equator and equinox to mean equator and equinox of date by applying precession.

6. The matrix $\mathbf{BP}$ transforms vectors from GCRS to mean equator and equinox of date by applying frame bias then precession. It is the product $\mathbf{P}\mathbf{B}$.

7. The matrix $\mathbf{N}$ transforms vectors from mean equator and equinox of date to true equator and equinox of date by applying the nutation (luni-solar + planetary).

8. The matrix $\mathbf{BPN}$ transforms vectors from GCRS to true equator and equinox of date. It is the product $\mathbf{NPB}$, applying frame bias, precession and nutation in that order.

9. The $X, Y, Z$ coordinates of the IAU 2006A Celestial Intermediate Pole are elements (3,1-3) of the GCRS-to-true matrix $\mathbf{BPN}$.

REFERENCE:

CALL:  
CALL iau_PNM00A ( DATE1, DATE2, RBPN )

ACTION:  
Form the matrix of precession-nutation for a given date (including frame bias), equinox based, IAU 2000A model.

GIVEN:  
\begin{align*}
DATE1 & \quad d \text{ TT as a two-part} \\
DATE2 & \quad d \quad \text{... Julian Date (Note 1)}
\end{align*}

RETURNED:  
RBPN \quad d(3,3) \quad \text{classical bias-precession-nutation matrix BPN (Note 2)}

NOTES:  
1. The TT date $DATE1+DATE2$ is a Julian Date, apportioned in any convenient way between the arguments $DATE1$ and $DATE2$. For example, $JD(TT) = 2450123.7$ could be expressed in any of these ways, among others:
\begin{align*}
DATE1 & \quad DATE2 \\
2450123.7D0 & \quad 0D0 \quad \text{(JD method)} \\
2451545.0D0 & \quad -1421.3D0 \quad \text{(J2000 method)} \\
2400000.5D0 & \quad 50123.2D0 \quad \text{(MJD method)} \\
2450123.5D0 & \quad 0.2D0 \quad \text{(date & time method)}
\end{align*}
The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The matrix operates in the sense
\[ v_{date} = BPN \cdot v_{GCRS} , \]
where the p-vector $v_{date}$ is with respect to the true equatorial triad of date $DATE1+DATE2$ and the p-vector $v_{GCRS}$ is with respect to the Geocentric Celestial Reference System (IAU, 2000).

3. A faster, but slightly less accurate, result (about 1 mas) can be obtained by using instead the iau_PNM00B routine.

REFERENCE:  
**CALL:**

CALL iau_PNM00B ( DATE1, DATE2, RBPN )

**ACTION:**

Form the matrix of precession-nutation for a given date (including frame bias), equinox based, IAU 2000B model.

**GIVEN:**

\[ DATE1 \text{ d} \quad \text{TT as a two-part,}\]
\[ DATE2 \text{ d} \quad \ldots \text{Julian Date (Note 1)} \]

**RETURNED:**

\[ RBPN \quad \text{d(3,3)} \quad \text{classical bias-precession-nutation matrix BPN (Note 2)} \]

**NOTES:**

1. The TT date \( DATE1+DATE2 \) is a Julian Date, apportioned in any convenient way between the arguments \( DATE1 \) and \( DATE2 \). For example, \( JD(TT) = 2450123.7 \) could be expressed in any of these ways, among others:

\[
\begin{array}{ccc}
\text{DATE1} & \text{DATE2} \\
2450123.7D0 & 0D0 & (\text{JD method}) \\
2451545.0D0 & -1421.3D0 & (J2000 method) \\
2400000.5D0 & 50123.2D0 & (MJD method) \\
2450123.5D0 & 0.2D0 & (date & time method)
\end{array}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The matrix operates in the sense

\[ \mathbf{v}_{\text{date}} = \mathbf{BPN} \mathbf{v}_{\text{GCRS}} \]

where the p-vector \( \mathbf{v}_{\text{date}} \) is with respect to the true equatorial triad of date \( DATE1+DATE2 \) and the p-vector \( \mathbf{v}_{\text{GCRS}} \) is with respect to the Geocentric Celestial Reference System (IAU, 2000).

3. The present routine is faster, but slightly less accurate (about 1 mas), than the iau_PNM00A routine.

**REFERENCE:**

CALL:
CALL iau_PNM06A ( DATE1, DATE2, RBPN )

ACTION:
Form the matrix of precession-nutation for a given date (including frame bias), equinox-based, IAU 2006 precession and IAU 2000A nutation models.

GIVEN:

\[
\begin{align*}
\text{DATE1} & \quad \text{d} \quad \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} \quad \text{... Julian Date (Note 1)}
\end{align*}
\]

RETURNED:

\[
\begin{align*}
\text{RBPN} & \quad \text{d(3,3)} \quad \text{classical bias-precession-nutation matrix BPN (Note 2)}
\end{align*}
\]

NOTES:

1. The TT date \text{DATE1+DATE2} is a Julian Date, apportioned in any convenient way between the arguments \text{DATE1} and \text{DATE2}. For example, \text{JD(TT)} = 2450123.7 could be expressed in any of these ways, among others:

\[
\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 \quad \text{(JD method)} \\
2451545.0D0 & \quad -1421.3D0 \quad \text{(J2000 method)} \\
2400000.5D0 & \quad 50123.2D0 \quad \text{(MJD method)} \\
2450123.5D0 & \quad 0.2D0 \quad \text{(date & time method)}
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The matrix operates in the sense

\[
v_{\text{date}} = \text{BPN} \cdot v_{\text{GCRS}},
\]

where the p-vector \(v_{\text{date}}\) is with respect to the true equatorial triad of date \text{DATE1+DATE2} and the p-vector \(v_{\text{GCRS}}\) is with respect to the Geocentric Celestial Reference System (IAU, 2000).

REFERENCES:

CALL : 

CALL iau_PNM80 ( DATE1, DATE2, RBPN )

ACTION : 

Form the matrix of precession-nutation for a given date, equinox based, IAU 1976 precession, IAU 1980 nutation models.

GIVEN : 

DATE1 d TT as a two-part...
DATE2 d ...Julian Date (Note 1)

RETURNED : 

RBPN d(3,3) classical precession-nutation matrix PN (Note 2)

NOTES :

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

\[
\begin{array}{ccc}
\text{DATE1} & \text{DATE2} \\
2450123.7D0 & 0D0 & \text{(JD method)} \\
2451545.0D0 & -1421.3D0 & \text{(J2000 method)} \\
2400000.5D0 & 50123.2D0 & \text{(MJD method)} \\
2450123.5D0 & 0.2D0 & \text{(date & time method)}
\end{array}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The matrix operates in the sense

\[ \mathbf{v}_{\text{date}} = \mathbf{PN} \mathbf{v}_{\text{J2000}} , \]

where the p-vector \( \mathbf{v}_{\text{date}} \) is with respect to the true equatorial triad of date DATE1+DATE2 and the p-vector \( \mathbf{v}_{\text{J2000}} \) is with respect to the mean equatorial triad of epoch J2000.0.

REFERENCE : 

CALL:
CALL iau_POM00 (XP, YP, SP, RPOM)

ACTION:
Form the matrix of polar motion for a given date, IAU 2000.

GIVEN:
XP,YP d coordinates of the pole (radians, Note 1)
SP d the TIO locator \( s' \) (radians, Note 2)

RETURNED:
RPOM d(3,3) polar-motion matrix \( W \) (Note 3)

NOTES:
1. XP and YP are the coordinates (in radians) of the Celestial Intermediate Pole with respect to the International Terrestrial Reference System (see IERS Conventions 2003), measured along the meridians 0° and 90° west respectively.

2. SP is the TIO locator \( s' \), in radians, which positions the Terrestrial Intermediate Origin on the equator. It is obtained from polar motion observations by numerical integration, and so is in principle unpredictable. However, it is dominated by a secular drift of about 47µas per century, and so can be taken into account by using \( s' = -47t \), where \( t \) is centuries since J2000.0. The routine iau_SP00 implements this approximation.

3. The matrix operates in the sense
\[
\mathbf{v}_{\text{TRS}} = \mathbf{W} \mathbf{v}_{\text{CIP}} ,
\]
meaning that it is the final rotation when computing the pointing direction to a celestial source for a terrestrial observer.

REFERENCE:
CALL:
CALL iau_PR00 ( DATE1, DATE2, DPSIPR, DEPSPR )

ACTION:
Precession-rate part of the IAU 2000 precession-nutation models (part of MHB2000).

GIVEN:

| DATE1 | d | TT as a two-part... |
| DATE2 | d | ...Julian Date (Note 1) |

RETURNED:

| DPSIPR | d | adjustment in longitude $\Delta \psi$ (radians, Notes 2,3) |
| DEPSPR | d | adjustment in obliquity $\Delta \epsilon$ (radians, Notes 2,3) |

NOTES:

1. The TT date DATE1+DATE2 is a Julian Date, apportioned in any convenient way between the arguments DATE1 and DATE2. For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:

<table>
<thead>
<tr>
<th>DATE1</th>
<th>DATE2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2450123.7D0</td>
<td>0D0</td>
</tr>
<tr>
<td>2451545.0D0</td>
<td>−1421.3D0</td>
</tr>
<tr>
<td>2400000.5D0</td>
<td>50123.2D0</td>
</tr>
<tr>
<td>2450123.5D0</td>
<td>0.2D0</td>
</tr>
</tbody>
</table>

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The precession adjustments are expressed as “nutation components”, corrections in longitude and obliquity with respect to the J2000.0 equinox and ecliptic.

3. Although the MHB2000 precession adjustments ($\Delta \psi, \Delta \epsilon$) are stated to be with respect to Lieske et al. (1977), the MHB2000 model does not specify which set of Euler angles are to be used and how the adjustments are to be applied. The most literal and straightforward procedure is to adopt the four-rotation

$$ P = R_3(\chi_A) R_1(-\omega_A) R_3(-\psi_A) R_1(\epsilon_0) $$

option, and to add $\Delta \psi$ to $\psi_A$ and $\Delta \epsilon$ to both $\omega_A$ and $\epsilon_A$.

4. This is an implementation of one aspect of the IAU 2000A nutation model, formally adopted by the IAU General Assembly in 2000, namely MHB2000 (Mathews et al. 2002).
REFERENCES:


CALL:
CALL iau_PREC76 ( DATE01, DATE02, DATE11, DATE12, ZETA, Z, THETA )

ACTION:
IAU 1976 precession model.

GIVEN:

\begin{align*}
&DATE01 \quad d \quad \text{start TT as a two-part...} \\
&DATE02 \quad d \quad \ldots\text{Julian Date (Note 1)} \\
&DATE11 \quad d \quad \text{end TT as a two-part...} \\
&DATE12 \quad d \quad \ldots\text{Julian Date (Note 1)}
\end{align*}

RETURNED:

\begin{align*}
&ZETA \quad d \quad \text{1st rotation: radians clockwise around } z \\
&Z \quad d \quad \text{3rd rotation: radians clockwise around } z \\
&THETA \quad d \quad \text{2nd rotation: radians counterclockwise around } y
\end{align*}

NOTES:

1. The dates DATE01+DATE02 and DATE11+DATE12 are Julian Dates, apportioned in any convenient way between the arguments DATEn1 and DATEn2. For example, JD(TT)=2450123.7 could be expressed in any of these ways, among others:

\begin{align*}
&\text{DATEn1} \quad \text{DATEn2} \\
&2450123.7\text{D0} \quad 0\text{D0} \quad \text{(JD method)} \\
&2451545.0\text{D0} \quad -1421.3\text{D0} \quad \text{(J2000 method)} \\
&2400000.5\text{D0} \quad 50123.2\text{D0} \quad \text{(MJD method)} \\
&2450123.5\text{D0} \quad 0.2\text{D0} \quad \text{(date & time method)}
\end{align*}

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience. The two dates may be expressed using different methods, but at the risk of losing some resolution.

2. The accumulated precession angles $\zeta$, $z$ and $\theta$ are expressed through canonical polynomials which are valid only for a limited time span. In addition, the IAU 1976 precession rate is known to be imperfect. The absolute accuracy of the present formulation is better than $0^\prime$1 from 1960 CE to 2040 CE, better than $1^\prime$ from 1640 CE to 2360 CE, and remains below $3^\prime$ for the whole of the period 500 CE to 3000 CE. The errors exceed $10^\prime$ outside the range 1200 CE to 3900 CE, exceed $100^\prime$ outside 4200 CE to 5600 CE and exceed $1000^\prime$ outside 6800 CE to 8200 CE.

3. The three angles are returned in the conventional order, which is not the same as the order of the corresponding Euler rotations. The precession matrix is given by

$$ P = R_3(-z) \ R_2(+\theta) \ R_3(-\zeta). $$

REFERENCE:

**CALL:**

CALL iau_RX ( PHI, R )

**ACTION:**

Rotate an r-matrix about the x-axis.

**GIVEN:**

PHI \( \phi \) (radians)

**GIVEN and RETURNED:**

\( R \) \( d(3,3) \) r-matrix, rotated

**NOTES:**

1. Calling this routine with positive \( \phi \) incorporates in the supplied r-matrix \( R \) an additional rotation, about the x-axis, anticlockwise as seen looking towards the origin from positive \( x \).

2. The additional rotation can be represented by this matrix:

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \phi + \sin \phi & \\
0 & -\sin \phi & \cos \phi \\
\end{pmatrix}
\]
iau_RXP  \quad \text{product of r-matrix and p-vector} \quad \text{iau_RXP}

CALL :
   CALL iau_RXP ( R, P, RP )

ACTION :
   Multiply a p-vector by an r-matrix.

GIVEN :
   \begin{align*}
   R & \quad \text{d(3,3)} & \quad \text{r-matrix} \\
   P & \quad \text{d(3)} & \quad \text{p-vector}
   \end{align*}

RETURNED :
   \begin{align*}
   RP & \quad \text{d(3)} & \quad R \times P
   \end{align*}
CALL:
CALL iau_RXR ( A, B, ATB )

ACTION:
Multiply two r-matrices.

GIVEN:
\[
\begin{align*}
A & \quad d(3,3) & \text{first r-matrix} \\
B & \quad d(3,3) & \text{second r-matrix}
\end{align*}
\]

RETURNED:
\[
\begin{align*}
ATB & \quad d(3,3) & A \times B
\end{align*}
\]
**iau_RY**

*rotate r-matrix about y axis*

**CALL:**

CALL iau_RY ( THETA, R )

**ACTION:**

Rotate an r-matrix about the y-axis.

**GIVEN:**

\[ THETA \quad \text{d} \quad \text{angle } \theta \text{ (radians)} \]

**GIVEN and RETURNED:**

\[ R \quad \text{d}(3,3) \quad \text{r-matrix, rotated} \]

**NOTES:**

1. Calling this routine with positive \( \theta \) incorporates in the supplied r-matrix \( R \) an additional rotation, about the y-axis, anticlockwise as seen looking towards the origin from positive y.

2. The additional rotation can be represented by this matrix:

\[
\begin{pmatrix}
+ \cos \theta & 0 & - \sin \theta \\
0 & 1 & 0 \\
+ \sin \theta & 0 & + \cos \theta
\end{pmatrix}
\]
CALL:
CALL iau_RZ ( PSI, R )

ACTION:
Rotate an r-matrix about the z-axis.

GIVEN:

\[ \text{PSI} \quad \text{d} \quad \text{angle } \psi \text{ (radians)} \]

GIVEN and RETURNED:

\[ \text{R} \quad \text{d(3,3)} \quad \text{r-matrix, rotated} \]

NOTES:

1. Calling this routine with positive \( \psi \) incorporates in the supplied r-matrix \( \text{R} \) an additional rotation, about the \( z \)-axis, anticlockwise as seen looking towards the origin from positive \( z \).

2. The additional rotation can be represented by this matrix:

\[
\begin{pmatrix}
+ \cos \psi & + \sin \psi & 0 \\
- \sin \psi & + \cos \psi & 0 \\
0 & 0 & 1
\end{pmatrix}
\]
**iau_S00**  
*CIO locator s given CIP X,Y, IAU 2000A  

**CALL:**

\[ D = \text{iau}_{-}S00\ (\text{DATE1}, \text{DATE2}, X, Y) \]

**ACTION:**

The CIO locator \( s \), positioning the Celestial Intermediate Origin on the equator of the Celestial Intermediate Pole, given the CIP’s \( X,Y \) coordinates. Compatible with IAU 2000A precession-nutation.

**GIVEN:**

- **DATE1**  
  d  
  TT as a two-part...
- **DATE2**  
  d  
  …Julian Date (Note 1)
- **X,Y**  
  d  
  CIP coordinates (Note 3)

**RETURNED** (function value):

\[ d \]

The CIO locator \( s \) in radians (Note 2)

**NOTES:**

1. The TT date \( \text{DATE1+DATE2} \) is a Julian Date, apportioned in any convenient way between the arguments **DATE1** and **DATE2**. For example, \( \text{JD(TT)} = 2450123.7 \) could be expressed in any of these ways, among others:

\[
\begin{array}{ccc}
\text{DATE1} & \text{DATE2} \\
2450123.7D0 & 0D0 & (\text{JD method}) \\
2451545.0D0 & -1421.3D0 & (\text{J2000 method}) \\
2400000.5D0 & 50123.2D0 & (\text{MJD method}) \\
2450123.5D0 & 0.2D0 & (\text{date & time method})
\end{array}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The CIO locator \( s \) is the difference between the right ascensions of the same point in two systems: the two systems are the GCRS and the CIP,CIO, and the point is the ascending node of the CIP equator. The quantity \( s \) remains below 0.‘1 throughout 1900-2100.

3. The series used to compute \( s \) is in fact for \( s + XY/2 \), where \( X \) and \( Y \) are the \( x \) and \( y \) components of the CIP unit vector; this series is more compact than a direct series for \( s \) would be. This routine requires \( X \) and \( Y \) to be supplied by the caller, who is responsible for providing values that are consistent with the supplied date.

4. The model is consistent with the IAU 2000A precession-nutation.
REFERENCES:


CALL:
\[ D = \text{iau\_S00A}(\text{DATE1}, \text{DATE2}, X, Y) \]

ACTION:
The CIO locator \( s \), positioning the Celestial Intermediate Origin on the equator of the Celestial Intermediate Pole, using the IAU 2000A precession-nutation model.

GIVEN:
- \( \text{DATE1} \) \( d \) TT as a two-part...
- \( \text{DATE2} \) \( d \) ...Julian Date (Note 1)

RETURNED (function value):
- \( d \) the CIO locator \( s \) in radians (Note 2)

NOTES:
1. The TT date \( \text{DATE1}+\text{DATE2} \) is a Julian Date, apportioned in any convenient way between the arguments \( \text{DATE1} \) and \( \text{DATE2} \). For example, \( \text{JD(TT)} = 2450123.7 \) could be expressed in any of these ways, among others:

   \[
   \begin{align*}
   \text{DATE1} & \quad \text{DATE2} \\
   2450123.7D0 & \quad 0D0 \quad (\text{JD method}) \\
   2451545.0D0 & \quad -1421.3D0 \quad (\text{J2000 method}) \\
   2400000.5D0 & \quad 50123.2D0 \quad (\text{MJD method}) \\
   2450123.5D0 & \quad 0.2D0 \quad (\text{date & time method})
   \end{align*}
   \]

   The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The CIO locator \( s \) is the difference between the right ascensions of the same point in two systems: the two systems are the GCRS and the CIP,CIO, and the point is the ascending node of the CIP equator. The quantity \( s \) remains below 0\(^\circ\)1 throughout 1900-2100.

3. The series used to compute \( s \) is in fact for \( s + XY/2 \), where \( X \) and \( Y \) are the \( x \) and \( y \) components of the CIP unit vector; this series is more compact than a direct series for \( s \) would be. The present routine uses the full IAU 2000A nutation model when predicting the CIP position. Faster results, with no significant loss of accuracy, can be obtained via the routine \text{iau\_S00B}, which uses instead the IAU 2000B truncated model.

REFERENCES:
CALL:
\[ D = \text{iau}\_\text{S00B} \left( \text{DATE1}, \text{DATE2}, X, Y \right) \]

ACTION:
The CIO locator \( s \), positioning the Celestial Intermediate Origin on the equator of the Celestial Intermediate Pole, using the IAU 2000B precession-nutation model.

GIVEN:
\[ \begin{align*}
\text{DATE1} & \quad \text{d} \quad \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} \quad \ldots \text{Julian Date (Note 1)}
\end{align*} \]

RETURNED (function value):
\[ \begin{align*}
d & \quad \text{the CIO locator} \ s \text{ in radians (Note 2)}
\end{align*} \]

NOTES:
1. The TT date \( \text{DATE1}+\text{DATE2} \) is a Julian Date, apportioned in any convenient way between the arguments \( \text{DATE1} \) and \( \text{DATE2} \). For example, \( \text{JD(TT)} = 2450123.7 \) could be expressed in any of these ways, among others:
\[ \begin{array}{ccc}
\text{DATE1} & \text{DATE2} \\
2450123.7D0 & 0D0 & \text{(JD method)} \\
2451545.0D0 & -1421.3D0 & \text{(J2000 method)} \\
2400000.5D0 & 50123.2D0 & \text{(MJD method)} \\
2450123.5D0 & 0.2D0 & \text{(date & time method)} \\
\end{array} \]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The CIO locator \( s \) is the difference between the right ascensions of the same point in two systems: the two systems are the GCRS and the CIP,CIO, and the point is the ascending node of the CIP equator. The quantity \( s \) remains below \( 0.1'' \) throughout 1900-2100.

3. The series used to compute \( s \) is in fact for \( s + XY/2 \), where \( X \) and \( Y \) are the \( x \) and \( y \) components of the CIP unit vector; this series is more compact than a direct series for \( s \) would be. The present routine uses the IAU 2000B truncated nutation model when predicting the CIP position. The routine \text{iau}\_\text{S00A} uses instead the full IAU 2000A model, but with no significant increase in accuracy and at some cost in speed.

REFERENCES:
CALL:
\[
D = \textit{iau\_S06} \left( \text{DATE1}, \text{DATE2}, X, Y \right)
\]

ACTION:
The CIO locator \( s \), positioning the Celestial Intermediate Origin on the equator of the Celestial Intermediate Pole, given the CIP’s \( X,Y \) coordinates. Compatible with IAU 2006/2000A precession-nutation.

GIVEN:
- \( \text{DATE1} \): TT as a two-part...
- \( \text{DATE2} \): ...Julian Date (Note 1)
- \( X,Y \): CIP coordinates (Note 3)

RETURNED (function value):
- \( d \): The CIO locator \( s \) in radians (Note 2)

NOTES:
1. The TT date \( \text{DATE1}+\text{DATE2} \) is a Julian Date, apportioned in any convenient way between the arguments \( \text{DATE1} \) and \( \text{DATE2} \). For example, JD(TT) = 2450123.7 could be expressed in any of these ways, among others:
   - \( \text{DATE1} \) = 2450123.7D0 0D0 (JD method)
   - \( \text{DATE2} \) = 2451545.0D0 −1421.3D0 (J2000 method)
   - \( \text{DATE1} \) = 2400000.5D0 50123.2D0 (MJD method)
   - \( \text{DATE2} \) = 2450123.5D0 0.2D0 (date & time method)
   The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The CIO locator \( s \) is the difference between the right ascensions of the same point in two systems: the two systems are the GCRS and the CIP,CIO, and the point is the ascending node of the CIP equator. The quantity \( s \) remains below \( 0'1 \) throughout 1900-2100.

3. The series used to compute \( s \) is in fact for \( s + XY/2 \), where \( X \) and \( Y \) are the \( x \) and \( y \) components of the CIP unit vector; this series is more compact than a direct series for \( s \) would be. This routine requires \( X \) and \( Y \) to be supplied by the caller, who is responsible for providing values that are consistent with the supplied date.

4. The model is consistent with the “P03” precession (Capitaine et al. 2003), adopted by IAU 2006 Resolution 1, 2006, and the IAU 2000A nutation (with P03 adjustments).
REFERENCES:

CALL:
\[ D = \text{iau}_\text{S06A}(\text{DATE1}, \text{DATE2}, X, Y) \]

ACTION:
The CIO locator \( s \), positioning the Celestial Intermediate Origin on the equator of the Celestial Intermediate Pole, using the IAU 2006 precession and IAU 2000A nutation models.

GIVEN:
- \( \text{DATE1} \) \( \text{d} \) TT as a two-part...
- \( \text{DATE2} \) \( \text{d} \) ... Julian Date (Note 1)

RETURNED (function value):
- \( d \) the CIO locator \( s \) in radians (Note 2)

NOTES:
1. The TT date \( \text{DATE1} + \text{DATE2} \) is a Julian Date, apportioned in any convenient way between the arguments \( \text{DATE1} \) and \( \text{DATE2} \). For example, \( \text{JD(TT)} = 2450123.7 \) could be expressed in any of these ways, among others:
   - \( \text{DATE1} \) \( 2450123.7D0 \) \( \text{DATE2} \) \( 0D0 \) (JD method)
   - \( \text{DATE1} \) \( 2451545.0D0 \) \( \text{DATE2} \) \( -1421.3D0 \) (J2000 method)
   - \( \text{DATE1} \) \( 2400000.5D0 \) \( \text{DATE2} \) \( 50123.2D0 \) (MJD method)
   - \( \text{DATE1} \) \( 2450123.5D0 \) \( \text{DATE2} \) \( 0.2D0 \) (date & time method)

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2. The CIO locator \( s \) is the difference between the right ascensions of the same point in two systems: the two systems are the GCRS and the CIP,CIO, and the point is the ascending node of the CIP equator. The quantity \( s \) remains below \( 0.1'' \) throughout 1900-2100.

3. The series used to compute \( s \) is in fact for \( s + XY/2 \), where \( X \) and \( Y \) are the \( x \) and \( y \) components of the CIP unit vector; this series is more compact than a direct series for \( s \) would be. The present routine uses the full IAU 2000A nutation model when predicting the CIP position.

REFERENCES:
CALL:
\[ D = \text{iau}_{\text{SP00}}(\text{DATE1}, \text{DATE2}) \]

ACTION:
The TIO locator \( s' \), positioning the Terrestrial Intermediate Origin on the equator of the Celestial Intermediate Pole.

GIVEN:

<table>
<thead>
<tr>
<th>DATE1</th>
<th>( d )</th>
<th>TT as a two-part...</th>
</tr>
</thead>
<tbody>
<tr>
<td>DATE2</td>
<td>( d )</td>
<td>...Julian Date (Note 1)</td>
</tr>
</tbody>
</table>

RETURNED (function value):

\( d \) the TIO locator \( s' \) in radians (Note 2)

NOTES:

1. The TT date \( DATE1+DATE2 \) is a Julian Date, apportioned in any convenient way between the arguments \( DATE1 \) and \( DATE2 \). For example, \( JD(TT) = 2450123.7 \) could be expressed in any of these ways, among others:

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</table>

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The TIO locator \( s' \) is obtained from polar motion observations by numerical integration, and so is in essence unpredictable. However, it is dominated by a secular drift of about \( 47\mu \text{as} \) per century, which is the approximation evaluated by the present routine.

REFERENCE:

CALL:  
CALL iau_TR ( R, RT )

ACTION:  
Transpose an r-matrix.

GIVEN:  
\[ R \] d(3,3) r-matrix

RETURNED:  
\[ RT \] d(3,3) transpose
CALL:

CALL iau_XY06 ( DATE1, DATE2, X, Y )

ACTION:

X, Y coordinates of celestial intermediate pole from series based on IAU 2006 precession and IAU 2000A nutation.

GIVEN:

\[ DATE1 \quad d \quad \text{TT as a two-part...} \]
\[ DATE2 \quad d \quad \ldots \text{Julian Date (Note 1)} \]

RETURNED:

\[ X, Y \quad d \quad \text{CIP X, Y coordinates (Note 2)} \]

NOTES:

1. The TT date \( DATE1+DATE2 \) is a Julian Date, apportioned in any convenient way between the arguments \( DATE1 \) and \( DATE2 \). For example, \( \text{JD(TT)} = 2450123.7 \) could be expressed in any of these ways, among others:

\[
\begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 \quad (\text{JD method}) \\
2451545.0D0 & \quad -1421.3D0 \quad (\text{J2000 method}) \\
2400000.5D0 & \quad 50123.2D0 \quad (\text{MJD method}) \\
2450123.5D0 & \quad 0.2D0 \quad (\text{date & time method})
\end{align*}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The \( X, Y \) coordinates are those of the unit vector towards the celestial intermediate pole. They represent the combined effects of frame bias, precession and nutation.

3. The fundamental arguments used are as adopted in IERS Conventions (2003) and are from Simon \textit{et al.} (1994) and Souchay \textit{et al.} (1999).

4. This is an alternative to the angles-based method, via the SOFA routine \texttt{iau_FW2XY} and as used in \texttt{iau_XYS06A} for example. The two methods agree at the 1\( \mu \)as level (at present), a negligible amount compared with the intrinsic accuracy of the models. However, it would be unwise to mix the two methods (angles-based and series-based) in a single application.
REFERENCES:

**iau_XYS00A**  
*CIP X,Y and CIO locator s, IAU 2000A*  
**iau_XYS00A**

**CALL:**

```
CALL iau_XYS00A ( DATE1, DATE2, X, Y, S )
```

**ACTION:**

For a given TT date, compute the \( X, Y \) coordinates of the Celestial Intermediate Pole and the CIO locator \( s \), using the IAU 2000A precession-nutation model.

**GIVEN:**

- \( DATE1 \)  
  - \( d \)  
  - TT as a two-part...
- \( DATE2 \)  
  - \( d \)  
  - ...Julian Date (Note 1)

**RETURNED:**

- \( X, Y \)  
  - \( d \)  
  - CIP \( X, Y \) coordinates (Note 2)
- \( S \)  
  - \( d \)  
  - the CIO locator \( s \) (Note 3)

**NOTES:**

1. The TT date \( DATE1+DATE2 \) is a Julian Date, apportioned in any convenient way between the arguments \( DATE1 \) and \( DATE2 \). For example, \( JD(TT) = 2450123.7 \) could be expressed in any of these ways, among others:
   
<table>
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<tr>
<th>( DATE1 )</th>
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<tr>
<td>2450123.5D0</td>
<td>0.2D0</td>
</tr>
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</table>

   The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The \( X, Y \) coordinates are those of the unit vector towards the celestial intermediate pole. They represent the combined effects of frame bias, precession and nutation.

3. The CIO locator \( s \) (in radians) positions the Celestial Intermediate Origin on the equator of the CIP.

4. A faster, but slightly less accurate result (about 1 mas for \( X,Y \)), can be obtained by using instead the \texttt{iau_XYS00B} routine.

**REFERENCE:**

**iau_XYS00B**  
_CIP X, Y and CIO locator s, IAU 2000B_  

**CALL**

CALL iau_XYS00B ( DATE1, DATE2, X, Y, S )

**ACTION**

For a given TT date, compute the \( X, Y \) coordinates of the Celestial Intermediate Pole and the CIO locator \( s \), using the IAU 2000B precession-nutation model.

**GIVEN**

\[ \begin{align*}
\text{DATE1} & \quad \text{d} \quad \text{TT as a two-part...} \\
\text{DATE2} & \quad \text{d} \quad \ldots \text{Julian Date (Note 1)}
\end{align*} \]

**RETURNED**

\[ \begin{align*}
\text{X, Y} & \quad \text{d} \quad \text{CIP X, Y coordinates (Note 2)} \\
S & \quad \text{d} \quad \text{the CIO locator} \; s \; (\text{Note 3})
\end{align*} \]

**NOTES**

1. The TT date \( \text{DATE1+DATE2} \) is a Julian Date, apportioned in any convenient way between the arguments \( \text{DATE1} \) and \( \text{DATE2} \). For example, \( \text{JD(TT)} = 2450123.7 \) could be expressed in any of these ways, among others:

\[ \begin{align*}
\text{DATE1} & \quad \text{DATE2} \\
2450123.7D0 & \quad 0D0 \quad \text{(JD method)} \\
2451545.0D0 & \quad −1421.3D0 \quad \text{(J2000 method)} \\
2400000.5D0 & \quad 50123.2D0 \quad \text{(MJD method)} \\
2450123.5D0 & \quad 0.2D0 \quad \text{(date & time method)}
\end{align*} \]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date & time methods are both good compromises between resolution and convenience.

2. The \( X, Y \) coordinates are those of the unit vector towards the celestial intermediate pole. They represent the combined effects of frame bias, precession and nutation.

3. The CIO locator \( s \) (in radians) positions the Celestial Intermediate Origin on the equator of the CIP.

4. The present routine is faster, but slightly less accurate (about 1 mas in \( X, Y \)), than the iau_XYS00A routine.

**REFERENCE**

**CALL:**

CALL iau_XYS06A ( DATE1, DATE2, X, Y, S )

**ACTION:**

For a given TT date, compute the \(X, Y\) coordinates of the Celestial Intermediate Pole and the CIO locator \(s\), using the IAU 2006 precession and IAU 2000A nutation models.

**GIVEN:**

\(DATE1\ d\) TT as a two-part...
\(DATE2\ d\) ... Julian Date (Note 1)

**RETURNED:**

\(X, Y\ d\) CIP \(X, Y\) coordinates (Note 2)
\(S\ d\) the CIO locator \(s\) (Note 3)

**NOTES:**

1. The TT date \(DATE1 + DATE2\) is a Julian Date, apportioned in any convenient way between the arguments \(DATE1\) and \(DATE2\). For example, \(JD(TT) = 2450123.7\) could be expressed in any of these ways, among others:

\[
\begin{array}{ll}
DATE1 & DATE2 \\
2450123.7D0 & 0D0 \text{ (JD method)} \\
2451545.0D0 & -1421.3D0 \text{ (J2000 method)} \\
2400000.5D0 & 50123.2D0 \text{ (MJD method)} \\
2450123.5D0 & 0.2D0 \text{ (date \& time method)} \\
\end{array}
\]

The JD method is the most natural and convenient to use in cases where the loss of several decimal digits of resolution is acceptable. The J2000 method is best matched to the way the argument is handled internally and will deliver the optimum resolution. The MJD method and the date \& time methods are both good compromises between resolution and convenience.

2. The \(X, Y\) coordinates are those of the unit vector towards the celestial intermediate pole. They represent the combined effects of frame bias, precession and nutation.

3. The CIO locator \(s\) (in radians) positions the Celestial Intermediate Origin on the equator of the CIP.

4. Series-based solutions for generating \(X, Y\) are also available: see Capitaine \& Wallace (2006) and iau_XY06.

**REFERENCES:**

8 APPENDIX

Interrelationships between IAU 2006/2000A SOFA Fortran routines

The three diagrams set out SOFA’s IAU 2006/2000A procedures and the principal routines that each calls; a few frequently-used utility routines, such as iau_BPN2XY and iau_FW2M, are omitted in most cases in order to avoid over-complicating the diagram.

The first line of each box gives the name of the Fortran routine. Those routines classed as “canonical” have their names printed in red. Routines that are FUNCTION (as opposed to SUBROUTINE) subprograms are indicated by an equals sign at the start of the third line, that precedes the quantity being returned. The second line lists the input (given) arguments (green), where all dates are given as either TT or UT (i.e. UT1); however, all routines use two-argument Julian dates and fraction, either TT or UT1, as indicated. The third and following lines list the output (returned) arguments. All angular quantities are expressed in radians. Usual mathematical symbols have been used to denote the quantities, with matrices given in bold.

<table>
<thead>
<tr>
<th>Line</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Routine name: red implies canonical procedure.</td>
</tr>
<tr>
<td>2.</td>
<td>Input arguments green, separated by commas. Note; all TT and UT (i.e. UT1) arguments occupy two arguments.</td>
</tr>
<tr>
<td>3.</td>
<td>Output arguments, separated by commas. In the case of FUNCTION subprograms, the value returned is preceded by an equals (=) sign.</td>
</tr>
</tbody>
</table>

Quantities returned by canonical routines

1. Precession angles γ, φ, ψ, which include frame bias.
2. Bias, precession, and bias-precession matrices, B, P, PB.
3. Mean obliquity of date, εA.
4. Nutation in longitude and obliquity, Δφ, Δε.
5. Earth rotation angle, θ.
6. Greenwich mean sidereal time, GMST.
7. Locators for the CIO (s) and TIO (s').
8. Series evaluations for (X, Y) coordinates of the CIP.
9. Fundamental arguments, I, I', F, D, Ω, λMa, λN, λBn, λSn, λUr, λNe, and pA from IERS Conventions 2003. None of these routines appear in the charts.

Other quantities returned

11. (X, Y) of the CIP is calculated using bias, precession and nutation angles, SOFA’s chosen method, with the sole exception of iau_XY06.
12. Nutation matrix N.
13. GCRS to equinox and equator of date; bias, precession and nutation matrix NPB.
14. GCRS to Celestial Intermediate Reference System: matrix C.
15. Polar motion matrix, including s' W.
16. GCRS to ITRS matrix Q⁻¹.
17. Greenwich apparent sidereal time, GAST.
18. Equation of the equinoxes, EE.
19. Equation of the origins, EO.
20. Equinox precession angles, ζB, zB, θB, including frame bias.
Chart 1: The main routines for transforming from the GCRS to CIRS, TIRS and ITRS.

* Note: `iau_NUT06A` calls `iau_NUT00A`.
IAU 2006/2000A SOFA FORTRAN ROUTINES

Chart 2: Equinox based routines for calculating GAST and related quantities.

Chart 3: Routines for bias, precession and nutation; not all are shown in Chart 1.

* The arguments supplied by the user may be any of the three sets given for iau_FW2M.
IAU 2006/2000A SOFA FORTRAN ROUTINES

NOTES

Equinox based

\[ [\text{ITRS}] = W \ R_3(\text{GAST}) \ NPB \ [\text{GCRS}] \]

CIO based

\[ [\text{ITRS}] = W \ R_3(\theta) \ C \ [\text{GCRS}] \]
\[ = Q^{-1}[\text{GCRS}] \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Rotation from / to</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Equinox based</td>
</tr>
<tr>
<td>B</td>
<td>Bias</td>
<td>ICRS to mean equator and equinox of J2000.0.</td>
</tr>
<tr>
<td>P</td>
<td>Precession</td>
<td>J2000.0 to mean equator and equinox of date.</td>
</tr>
<tr>
<td>N</td>
<td>Nutation</td>
<td>Mean equator of date to true equator and equinox of date.</td>
</tr>
<tr>
<td>R₃(GAST)</td>
<td></td>
<td>True equator and equinox of date to the Terrestrial Intermediate Reference System.</td>
</tr>
<tr>
<td>NPB</td>
<td></td>
<td>GCRS → true of date</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CIO based</td>
</tr>
<tr>
<td>C</td>
<td>Bias, precession-nutation</td>
<td>GCRS to Celestial Intermediate Reference System (true equator of date).</td>
</tr>
<tr>
<td>R₃(θ)</td>
<td>Earth rotation angle</td>
<td>CIRS to the Terrestrial Intermediate Reference System.</td>
</tr>
<tr>
<td>Q⁻¹</td>
<td></td>
<td>GCRS to the International Terrestrial Reference System.</td>
</tr>
<tr>
<td>Q</td>
<td></td>
<td>ITRS → GCRS</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Terrestrial</td>
</tr>
<tr>
<td>W</td>
<td>Polar motion</td>
<td>TIRS to the International Terrestrial Reference System; includes R₃(s').</td>
</tr>
</tbody>
</table>